

***Proton Flows in Pb + Pb Collisions
at 40 A GeV
(Collective Flows in High-Energy Heavy-Ion
Collisions at SPS Energies)***

P. K. SAHU (Institute of Physics, Bhubaneswar)

Refs.

P. K. Sahu, Phys. Rev. C 77, 024904-1 (2008)

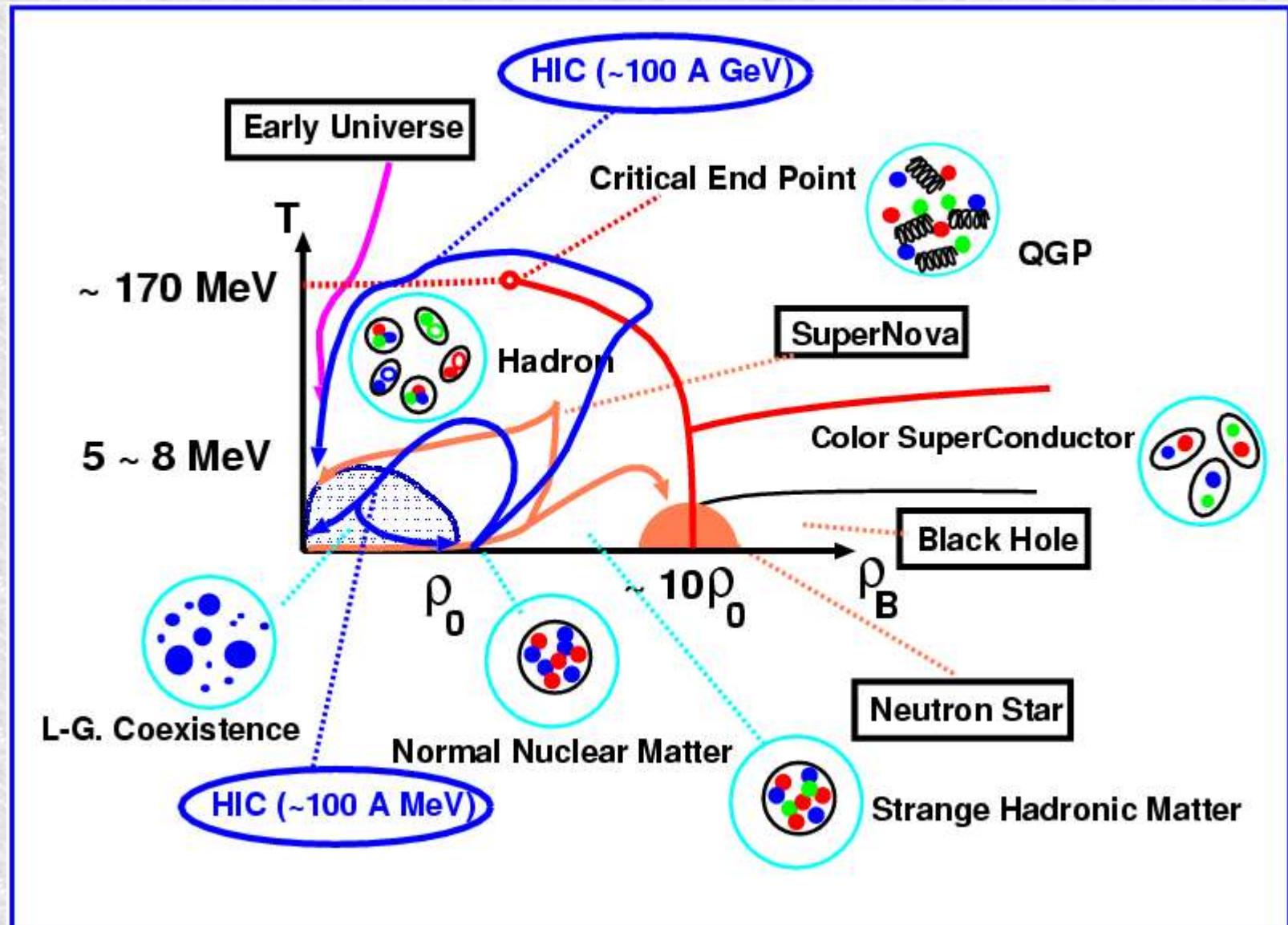
Isse, Ohnishi, Otuka, Nara, Sahu, Phys. Rev. C72, 064908 (2005)

Sahu and Cassing, Nucl. Phys. A712, 357 (2002)

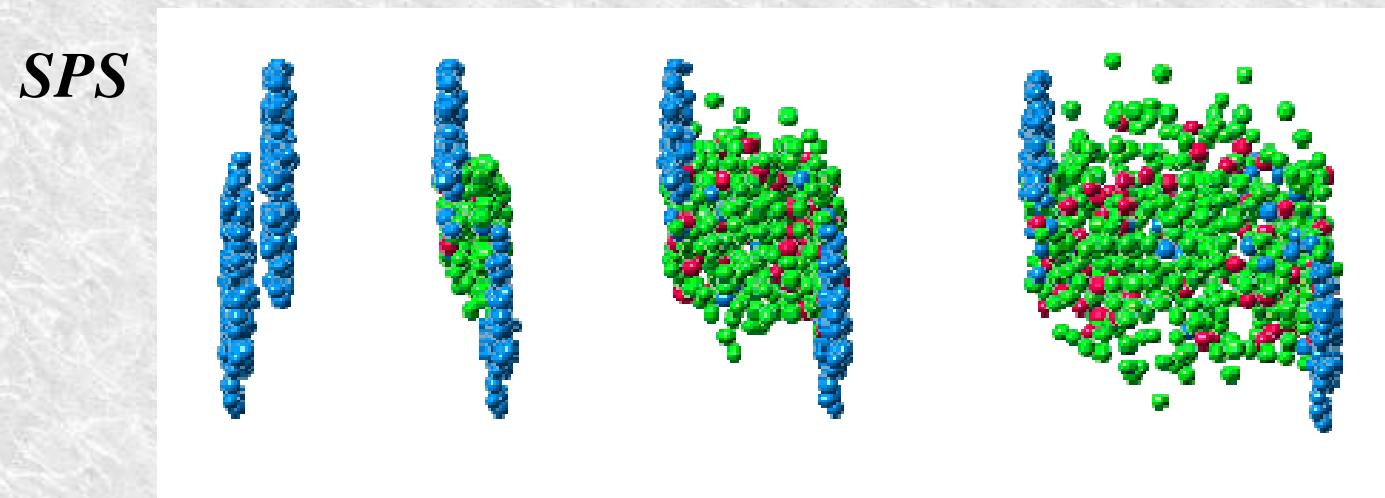
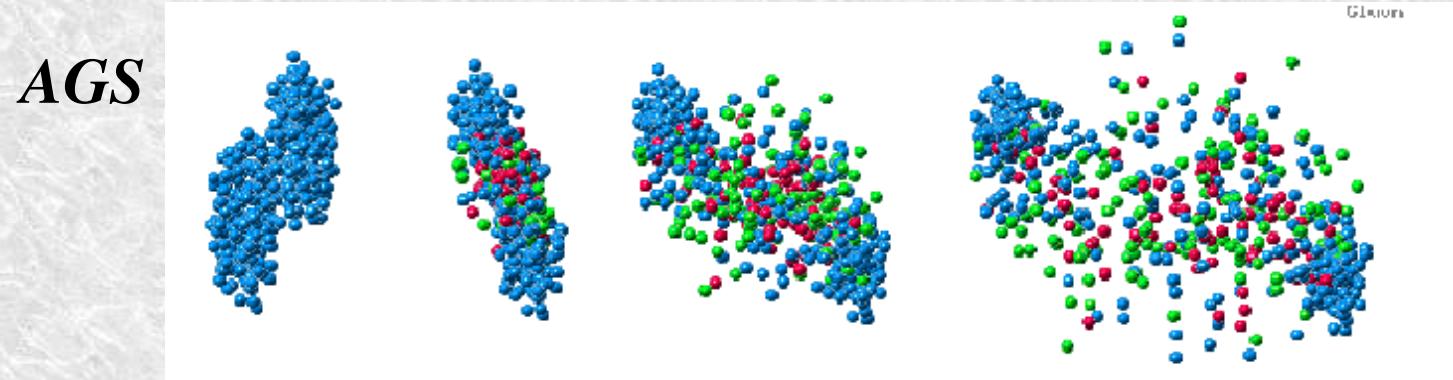
Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672, 376 (2000)

Sahu, Hombach, Cassing, Effenberger, Mosel, Nucl. Phys. A640, 493 (1998)

Hadronic Matter Phase Diagram

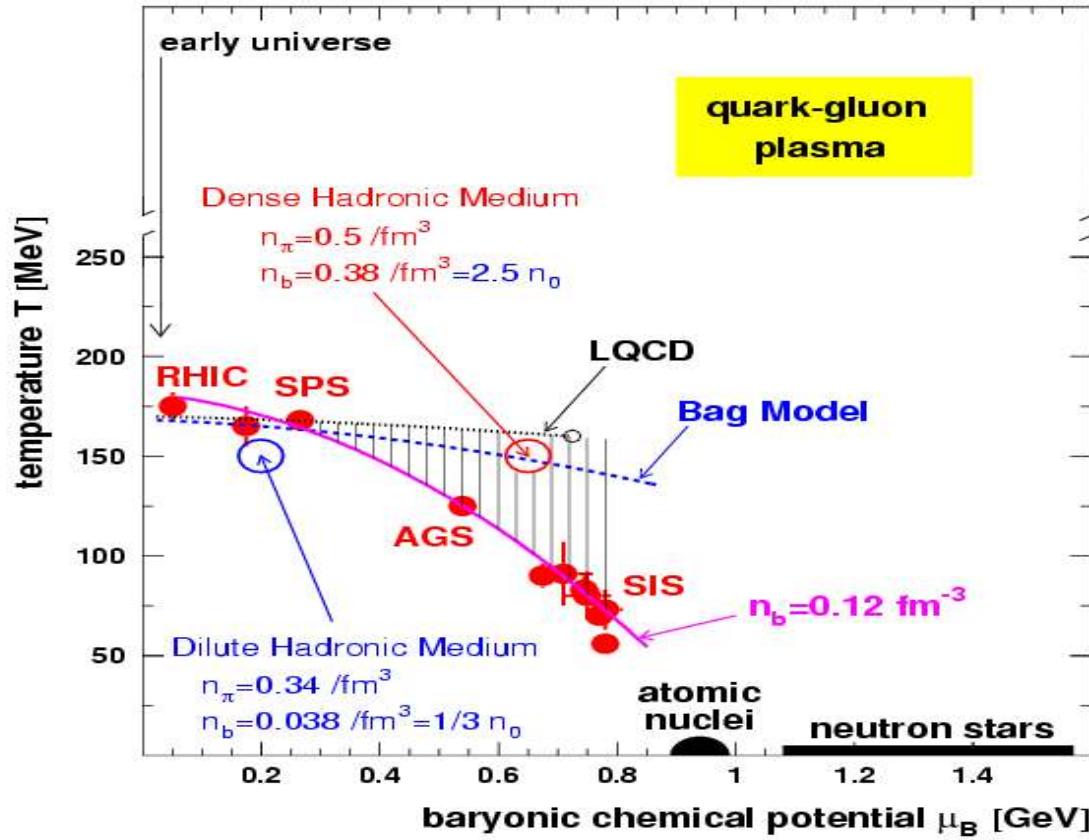


High T and/or High ρ Matter: High Energy Heavy-Ion Collisions



QCD Phase Transition at High T ($\mu = 0$)

HIC Experiment



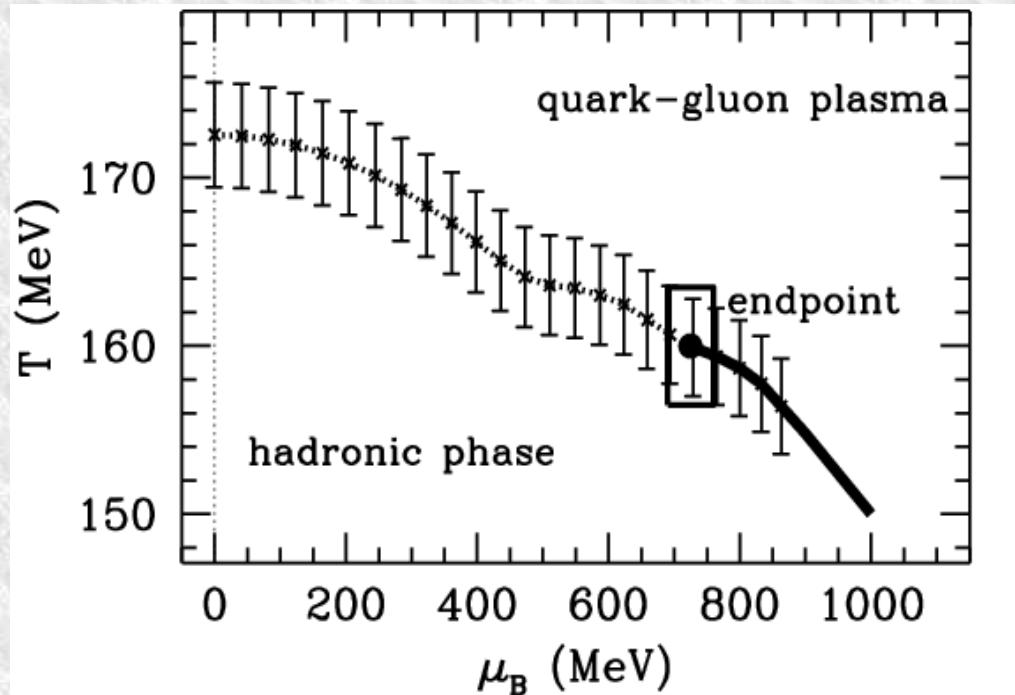
2002 (Braun-Munzinger et al.

J. Phys. G28 (2002) 1971.)

Chem. Freeze-Out \approx Phase Transition Boundary in LQCD

QCD Phase Transition at Finite μ

■ Full Lattice QCD



*Finite μ : Fodor & Katz,
JHEP 0203 (2002), 014.*

Zero Chem. Pot. : Cross Over (2nd order at Chiral Limit)

Finite Chem. Pot.: Critical End Point

Physics of Dense Nuclear Matter

* *Why is it interesting ?*

- *Phase transition/Composition Change by INTERACTION*
- *cf. Hot Nuclear Matter phase transition: # of DOF*
- *Full Lattice QCD is not (yet ?) possible at Large μ !*
- *Modification of Hadrons in Hot Medium*
- *Close relation to Compact Astrophysical Objects*

■ *How do we compress the Nucleus ?*

- ★ *Hot and Dense: High-Energy Heavy-Ion Collisions*
- ★ *Cold and Dense: ????? (In Neutron Stars)*

■ *What do we want to know in High-E. HI Collisions ?*

- ★ *QCD Phase Transition, Medium Effects on Hadrons*
- ★ *Equation of State*

Collective Flows

in High-Energy Heavy-Ion Collisions

What is Collective Flow ?

(Directed) Flow (dP_x/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

Thermalization
& Pressure Gradient

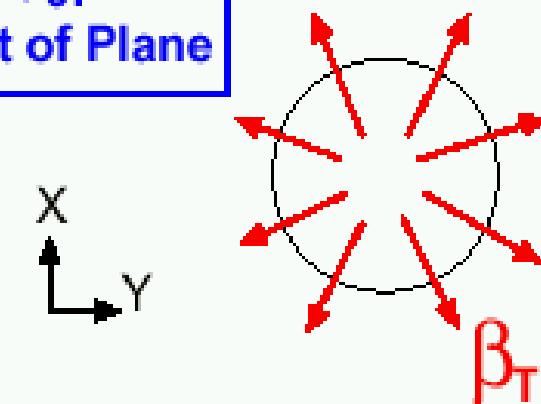
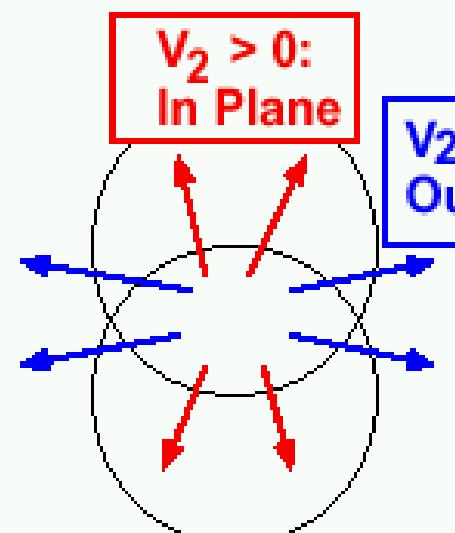
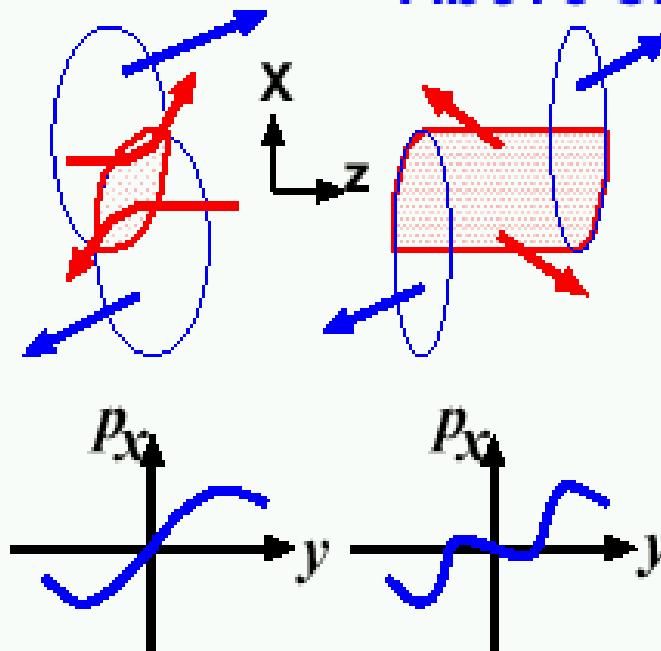
Radial Flow (β_T)

Pressure History

$$\epsilon \frac{DV}{Dt} = -\nabla P$$

$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

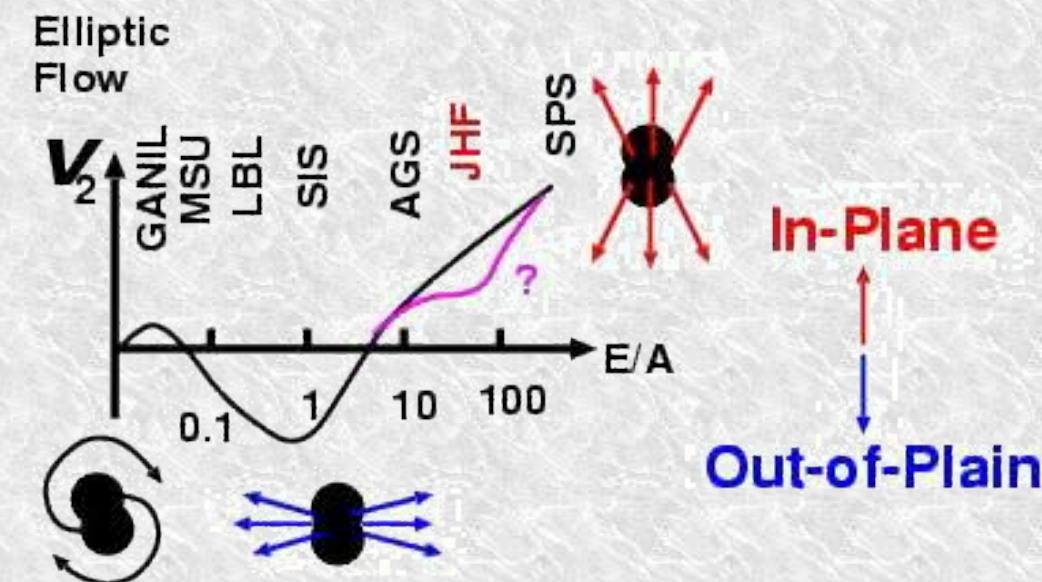
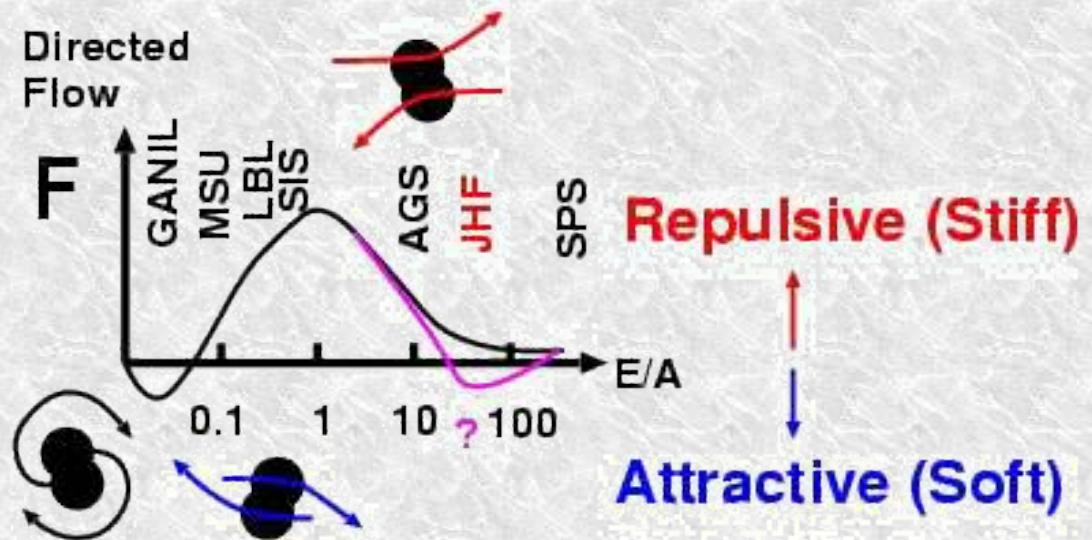
Until AGS Above SPS



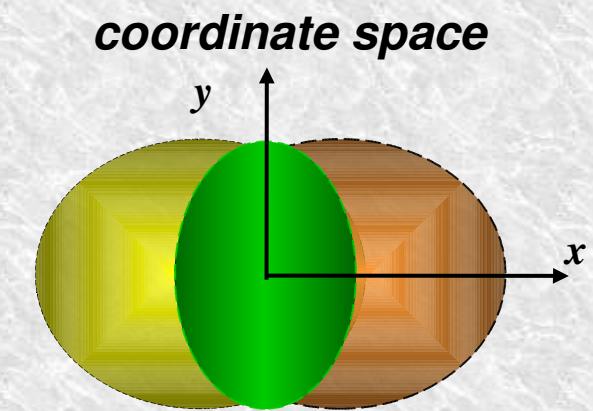
$$F = d \langle P_x \rangle / d(y/y_{nn})$$

$$v_2 = \langle \cos 2\varphi \rangle$$

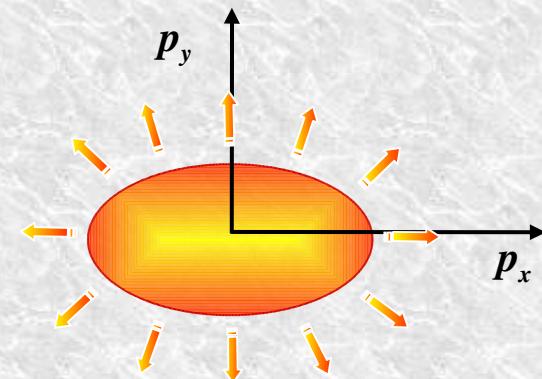
$$\frac{dN}{d\phi} = v_0(1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi))$$



Coordinate space configuration anisotropic (almond shape) however, initial momentum distribution isotropic (spherically symmetric)



Momentum space



Only interactions among constituents generate a pressure gradient, which transforms the initial coordinate space anisotropy into a momentum space anisotropy

Collective Flow and EOS: Old Problem ?

■ *1980's: First Suggestions and Measurement*

- ★ *Hydrodynamics suggested the Existence of Flow.*
- ★ *Strong Collective Flow suggests Hard EOS*

■ *1990's: Deeper Discussions in Wider Einc Range*

- ★ *Momentum Dep. Pot. can generate Strong Flows.*
- ★ *Einc deps. implies the importance of Momentum Deps.*
- ★ *Flow Measurement up to AGS Energies.*

■ *2000's: Extension to SPS and RHIC Energies*

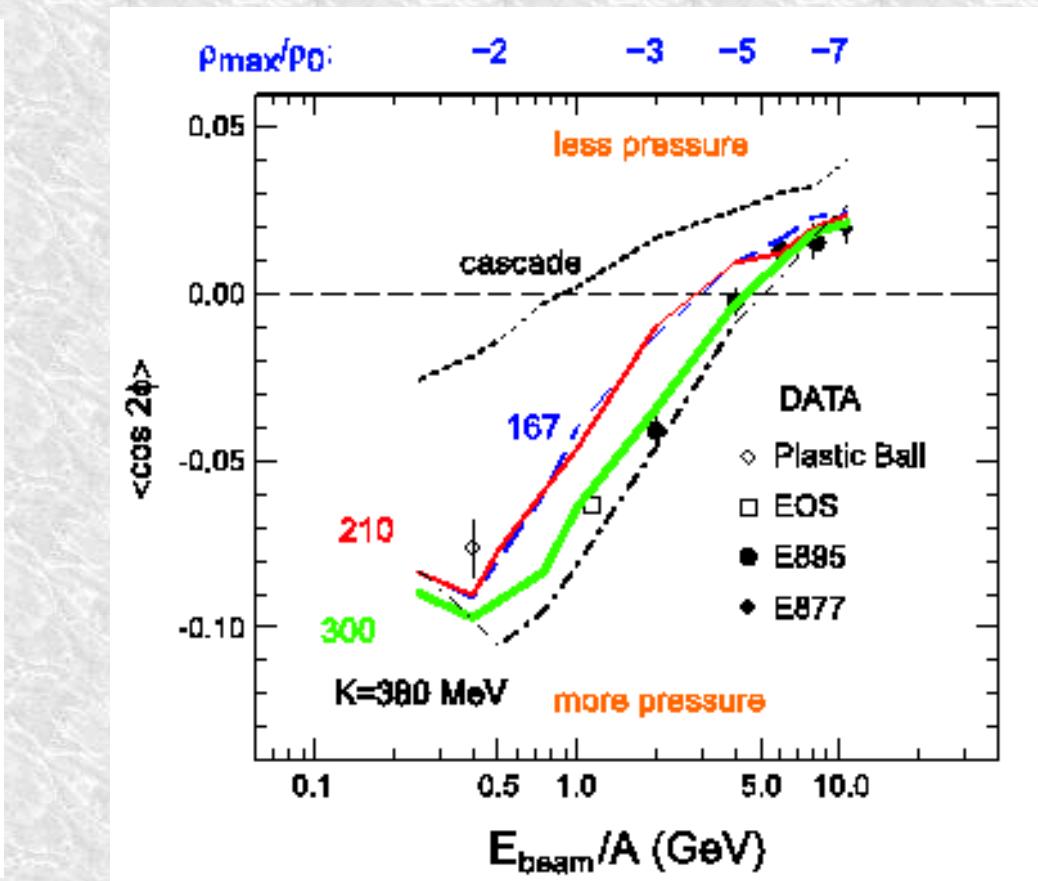
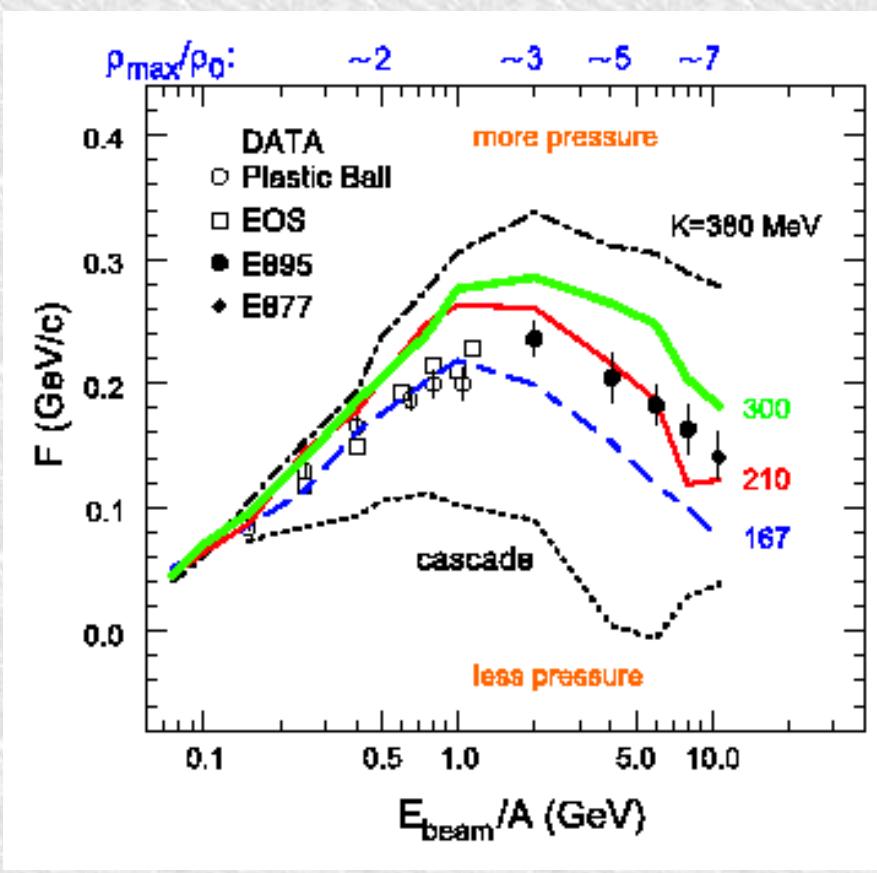
- ★ *EOS is determined with Mom. AND Density Dep. Pot. ?*

Old but New (Continuing) Problem !

Is the EOS determined ?

Collective Flows up to AGS energies

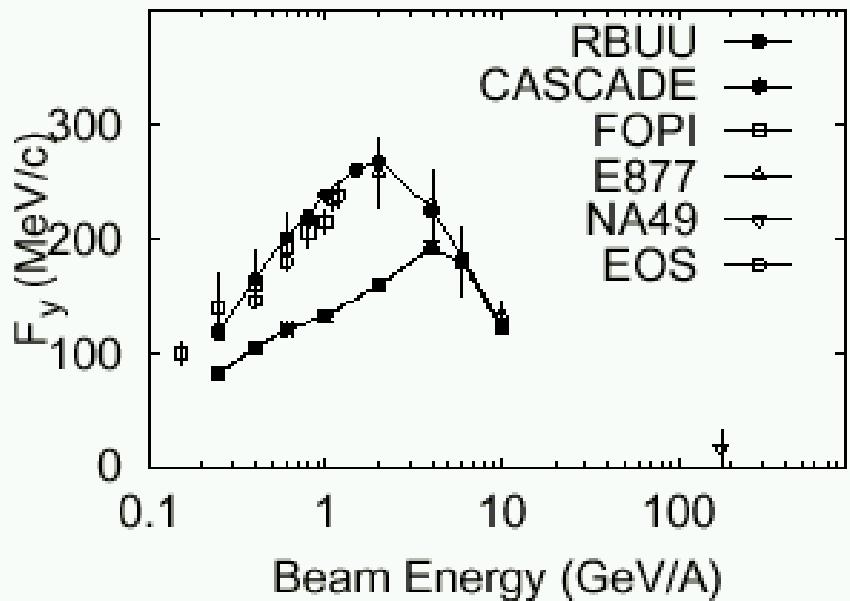
(P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592)



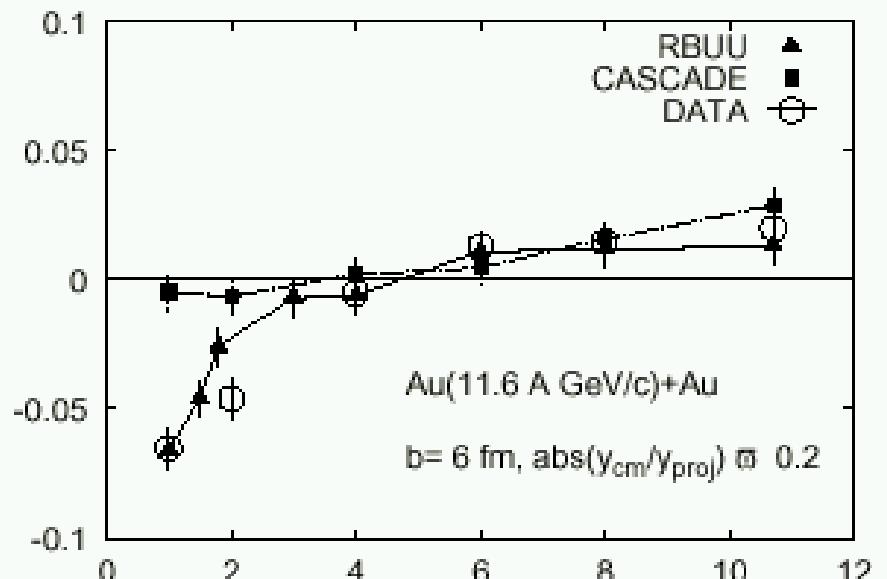
Our Results

(Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672 (2000), 376)

Sideward Flow



Elliptic Flow (v_2)



K=300 MeV for all the incident energies.

What should be done ? Flows at AGS and SPS

- Results are *MODEL DEPENDENT* at AGS energies
 - ★ *Consistent Understanding in wide inc. energy range is necessary*
- *Flows at Lower SPS energy (20-80 A GeV) have been measured recently.*
- *Not Very Seriously Investigated in relation to EOS*
 - ★ *High T but relatively Low Density (Mom. Dep. Potential)*
 - ★ *RQMD has been applied to SPS energies, but only with Mom. Indep. Potential.*

**We analyze Collective Flows at AGS and SPS energies
with Momentum Dependent Potentials**

Nonequilibrium Transport Model and the Equation of State

Hadronic Cascade Part: RBUU

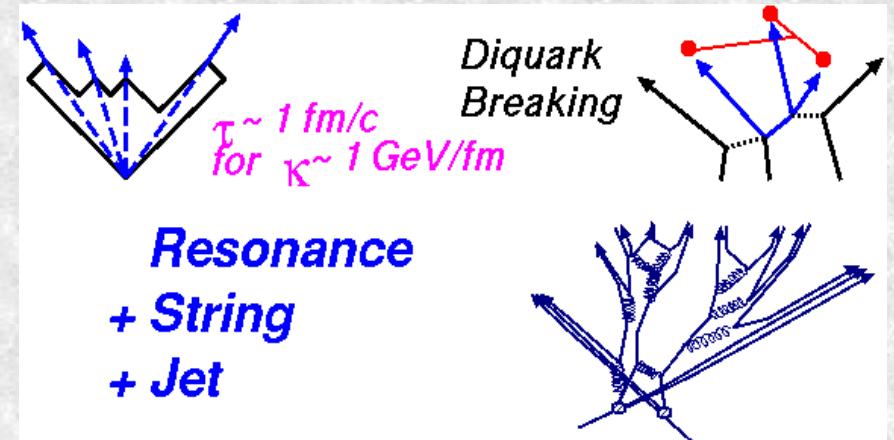
(Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672 (2000) 376.)

■ *Particle Degrees of Freedom*

- ★ *Hadrons (h , $m \sim 2$ GeV) + Strings (s)*

■ *Cross Sections*

- ★ *Hadronic ($hh \rightarrow hh$, $hh \rightarrow h$, $h \rightarrow hh$)*
- ★ *Soft ($hh \rightarrow s$, $hh \rightarrow ss$, $s \rightarrow hh$, $hh \rightarrow hs$ [1], $sh \rightarrow s'h$ [2])*



[1] "DPM + Lund" (~HIJING) + Phase Space [2] Constituent Rescattering (~RQMD)

Relativistic BUU(RBUU)

(*Sahu et al., Nucl. Phys. A640, (1998) 493 ;*
Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672 (2000) 376).

Constraints

Boltzmann equation for phase-space distribution f :

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_p}{\partial p} \frac{\partial f}{\partial \tau} - \frac{\partial \epsilon_p}{\partial \tau} \frac{\partial f}{\partial p} = I$$

$$\begin{aligned} & \{ (\Pi_\mu - \Pi_\nu \partial_\mu^p U_h^\nu - M_h^* \partial_\mu^p U_h^S) \partial_x^\mu + \\ & (\Pi_\nu \partial_\mu^x U_h^\nu + M_h^* \partial_\mu^x U_h^S) \partial_p^\mu \} f_h(x, p) \\ &= \sum_{h_2 h_3 h_4} \int d2 d3 d4 [G^\dagger G]_{12 \rightarrow 34} \\ & \quad \times \delta_\Gamma^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4) \\ & \quad \times \{ f_{h_3}(x, p_3) f_{h_4}(x, p_4) \bar{f}_h(x, p) \bar{f}_{h_2}(x, p_2) \\ & \quad - f_h(x, p) f_{h_2}(x, p_2) \bar{f}_{h_3}(x, p_3) \bar{f}_{h_4}(x, p_4) \} \end{aligned}$$

RBUU (cont.)

- $U_h^S(x, p)$ and $U_h^\mu(x, p)$ -real part of scalar and vector hadron self energies
- $[G^+ G]_{12 \rightarrow 34} \delta_\Gamma^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$ is the 'transition rate' for the process $1 + 2 \rightarrow 3 + 4$.

$$\begin{aligned} M_h^*(x, p) &= M_h + U_h^S(x, p) \\ \Pi^\mu(x, p) &= p^\mu - U_h^\mu(x, p) \end{aligned},$$

the phase-space factors

$$\bar{f}_h(x, p) = 1 - f_h(x, p)$$

In the mean field limit, from EOM

Vector Potential

$$U_v = g_\omega \omega_0 = \frac{g_\omega^2}{m_\omega^2} \rho_B$$

Scalar Potential

$$\begin{aligned} U_s &= g_\sigma \sigma_0 = m - m^* \\ &= \frac{g_\sigma^2}{m_\sigma^2} \rho_s - \frac{g_\sigma^2}{m_\sigma^2} b m (m - m^*)^2 \\ &\quad - \frac{g_\sigma^2}{m_\sigma^2} c ((m - m^*)^3) \end{aligned}$$

Choice of Potential

(Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672 (2000), 376)

$$\frac{\Lambda_s^2 - ap^2}{\Lambda_s^2 + p^2} \quad \text{and} \quad \frac{\Lambda_v^2 - bp^2}{\Lambda_v^2 + p^2}$$

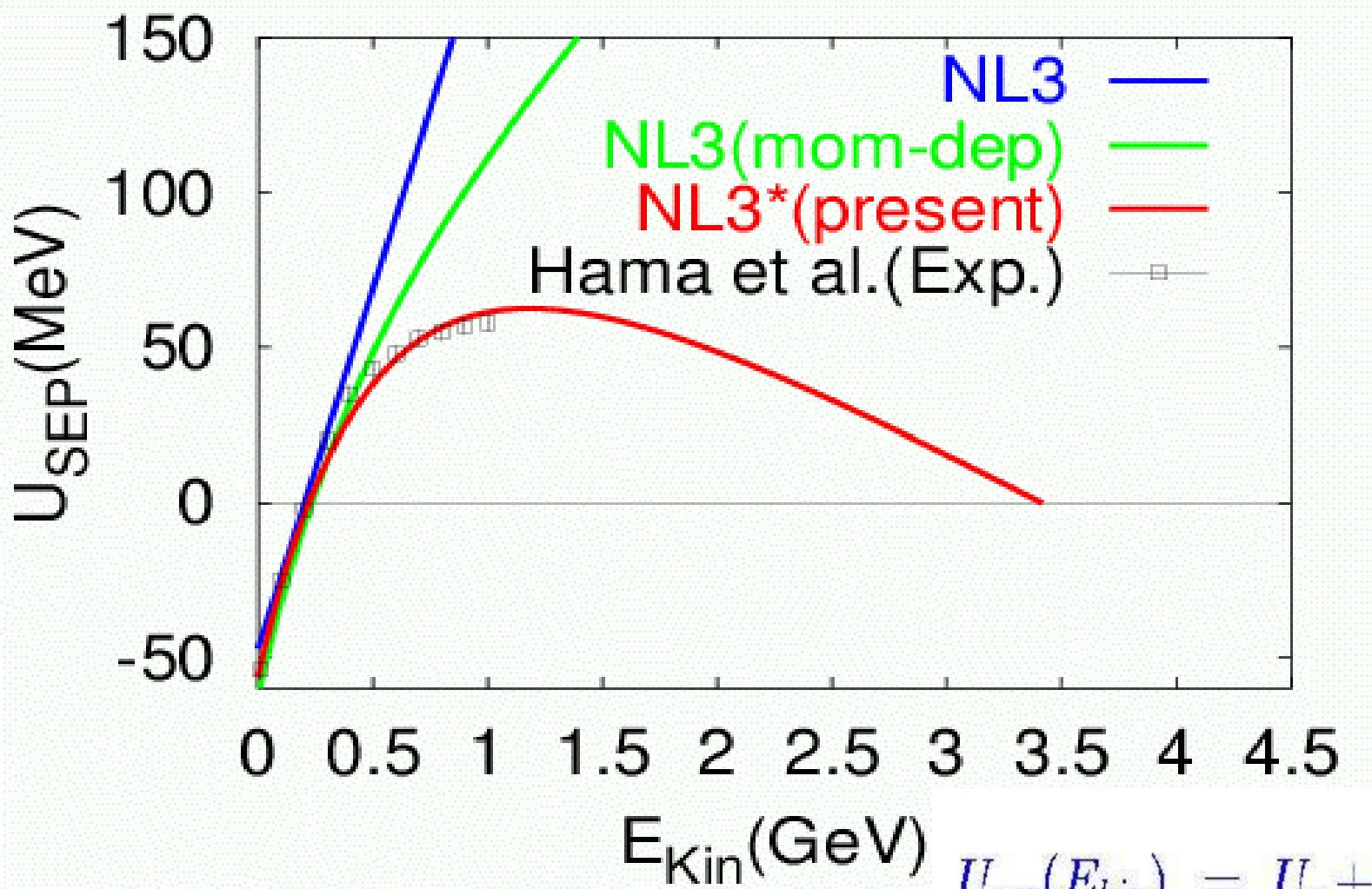
$$a=0.5 \text{ and } b=0.16$$

$$\Lambda s = 1.0 \text{ GeV} \quad \Lambda v = 0.9 \text{ GeV}$$

$$\begin{aligned} U_{sep}(E_{kin}) &= U_s + U_0 \\ &+ \frac{1}{2M} (U_s^2 - U_0^2) + \frac{U_0}{M} E_{kin} \end{aligned}$$

Momentum Dependence of N-A Potential

(Sahu, Cassing, Mosel, Ohnishi, Nucl. Phys. A672 (2000), 376)



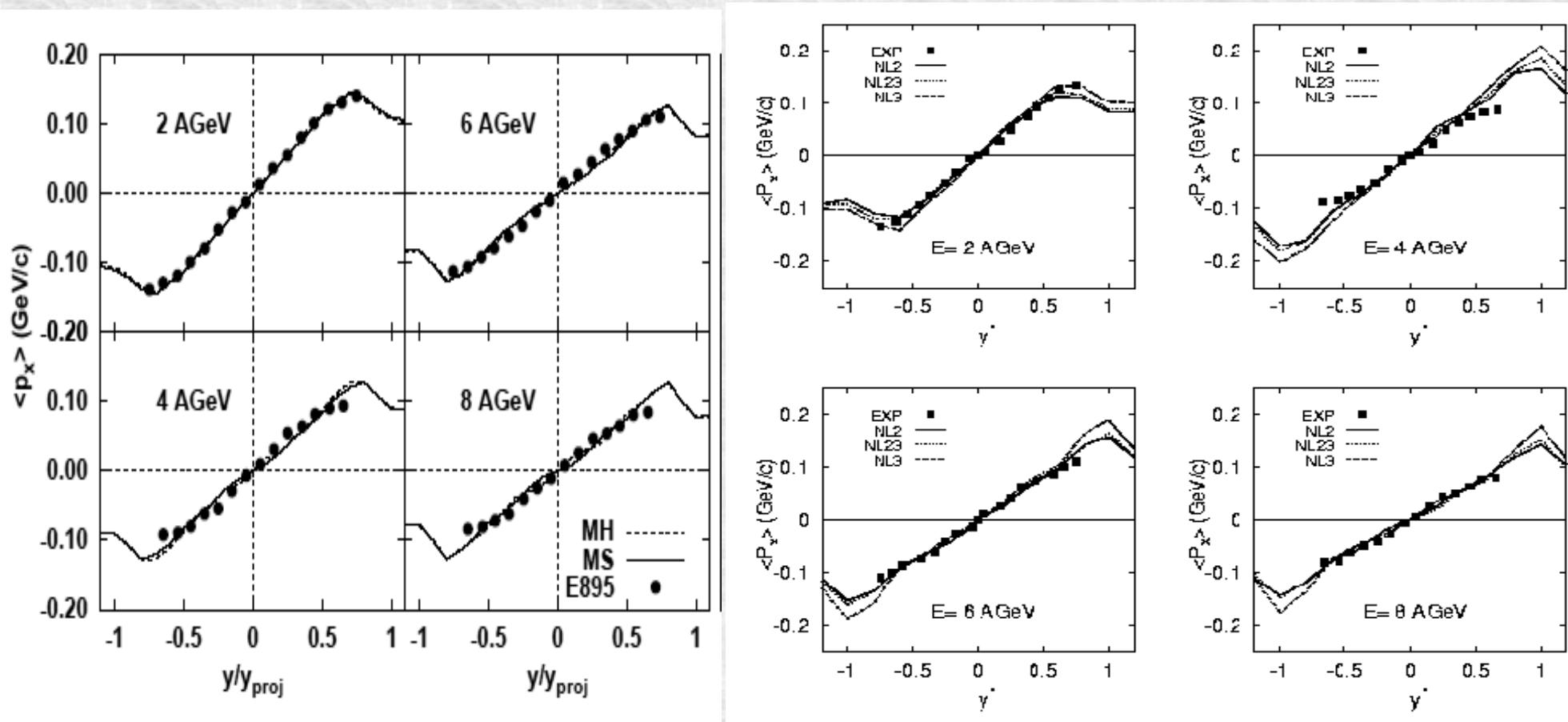
$$\begin{aligned} U_{sep}(E_{kin}) = & U_s + U_0 \\ & + \frac{1}{2M}(U_s^2 - U_0^2) + \frac{U_0}{M}E_{kin} \end{aligned}$$

Hadronic Model Study of Collective Flows from AGS to SPS Energies

Sideflow at AGS Energies

(Phys. Rev. C72, 064908 (2005); Nucl. Phys. A712, 357 (2002))

Mom. Dep. MF



Momentum Dep. Pot. generally give good description of Data.

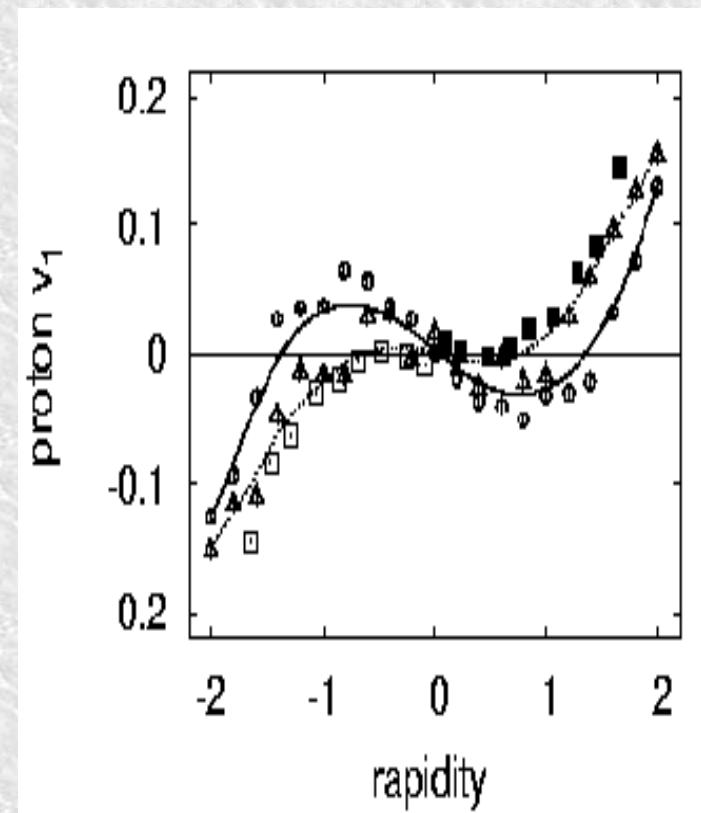
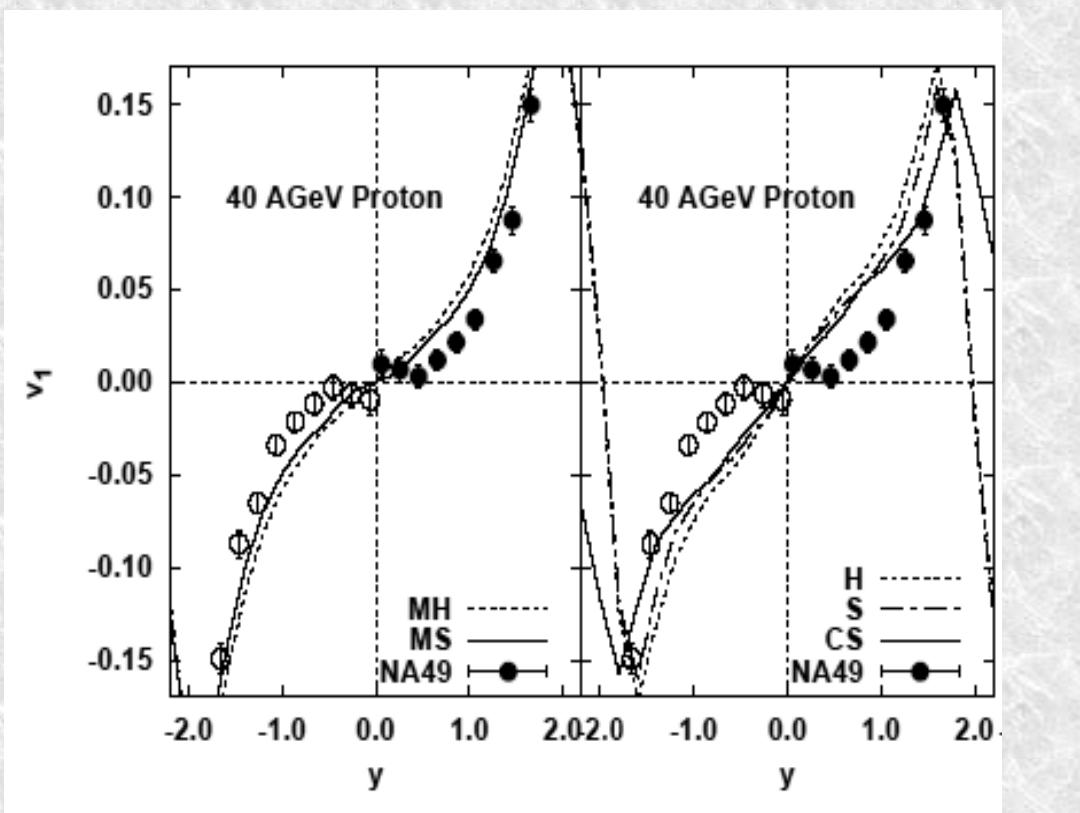
Directed at SPS Energies (I)

Phys. Rev. C72, 064908 (2005); Phys. Rev. C77, 024904-1 (2008)

$$v_1 \equiv \langle \cos \varphi \rangle = \langle p_x / p_T \rangle$$

Mom. Dep. Casc. & Mom. Indep.

Mom. Dep.& Casc.

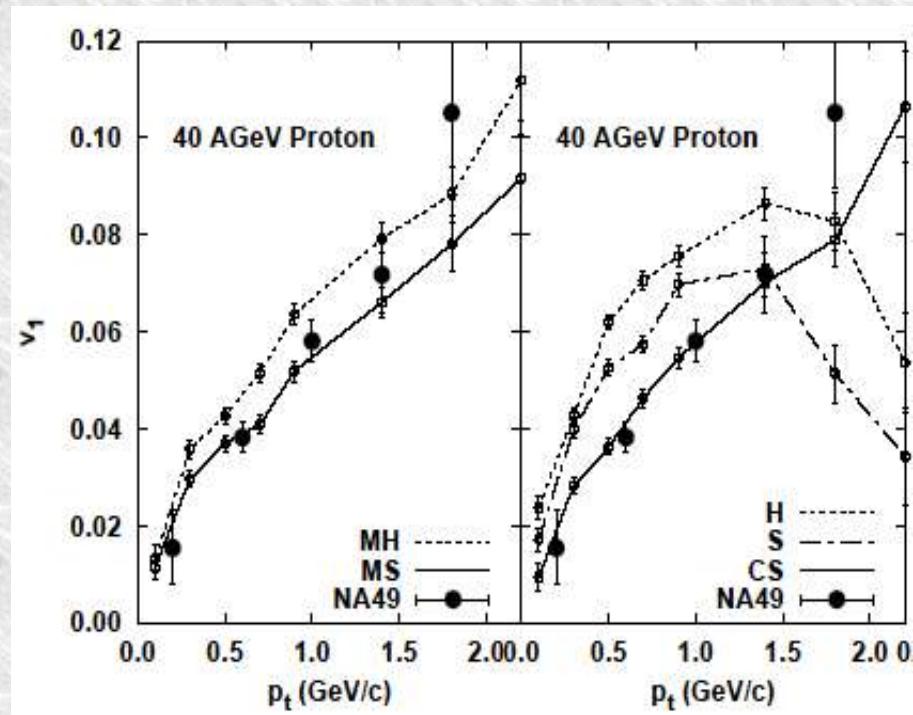


At 40 A GeV, Reverse Pressure from Spectator can be seen.

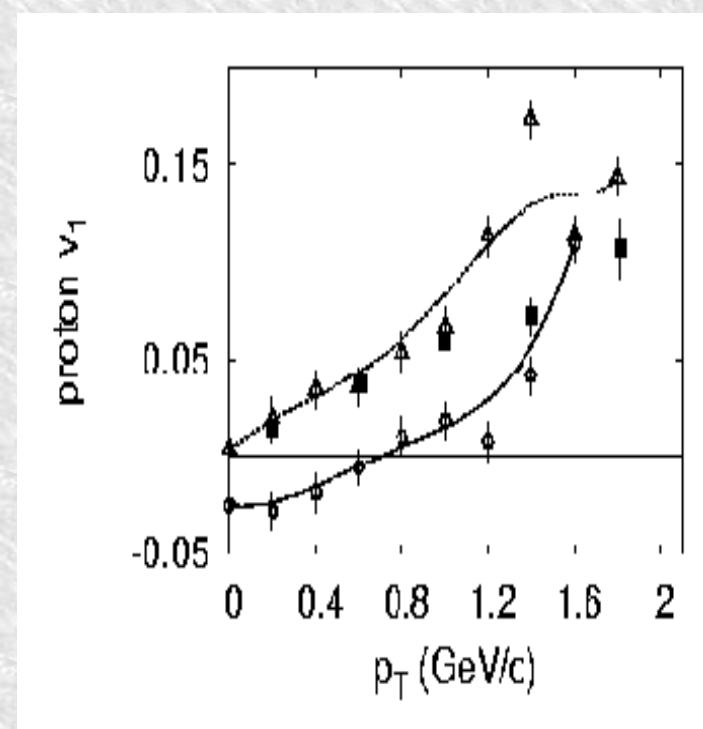
Directed Flow at SPS Energies (2)

(*Phys. Rev. C72, 064908 (2005); Phys. Rev. C77, 024904-1 (2008)*)

Mom. Dep. Casc. & Mom. Indep.



Mom. Dep. & Casc.

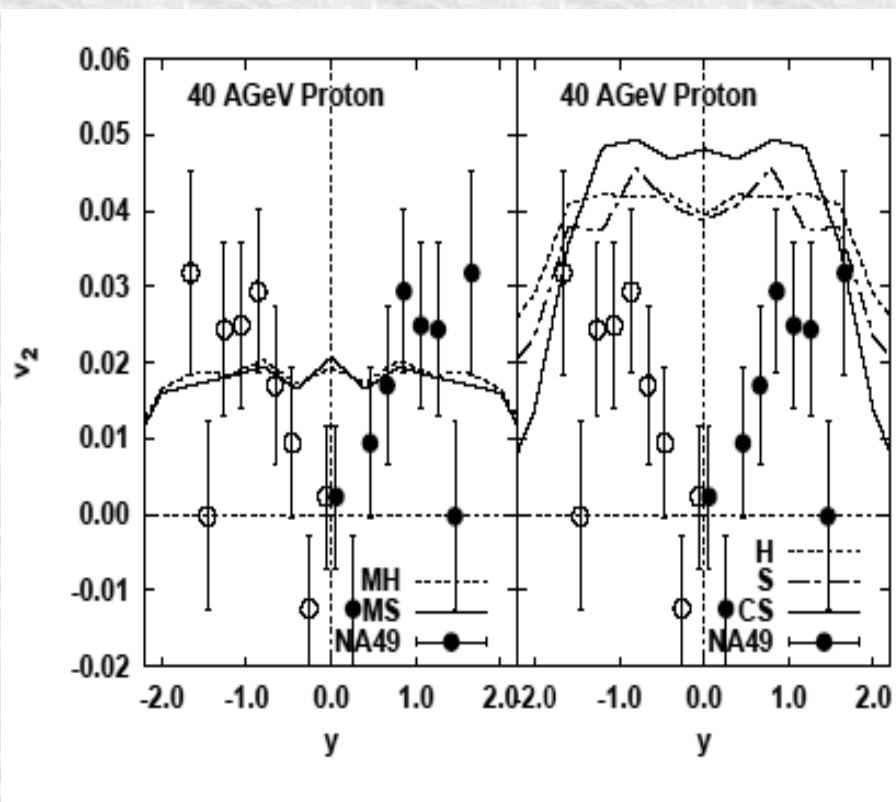


We find STIFFNESS dependence in $V1(Pt)$

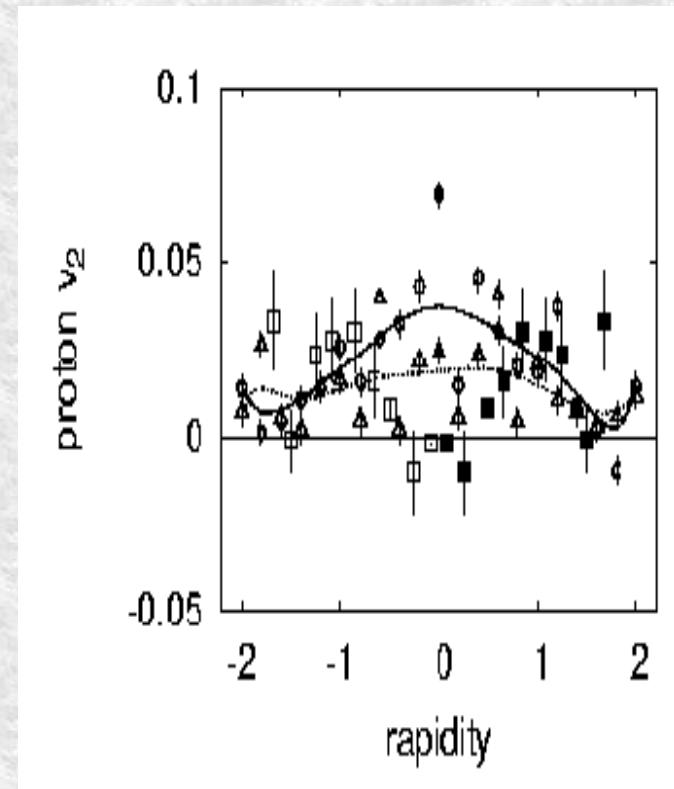
Elliptic Flow at SPS Energies (1)

(*Phys. Rev. C72, 064908 (2005); Phys. Rev. C77, 024904-1 (2008)*)

Mom. Dep. Casc. & Mom. Independ.



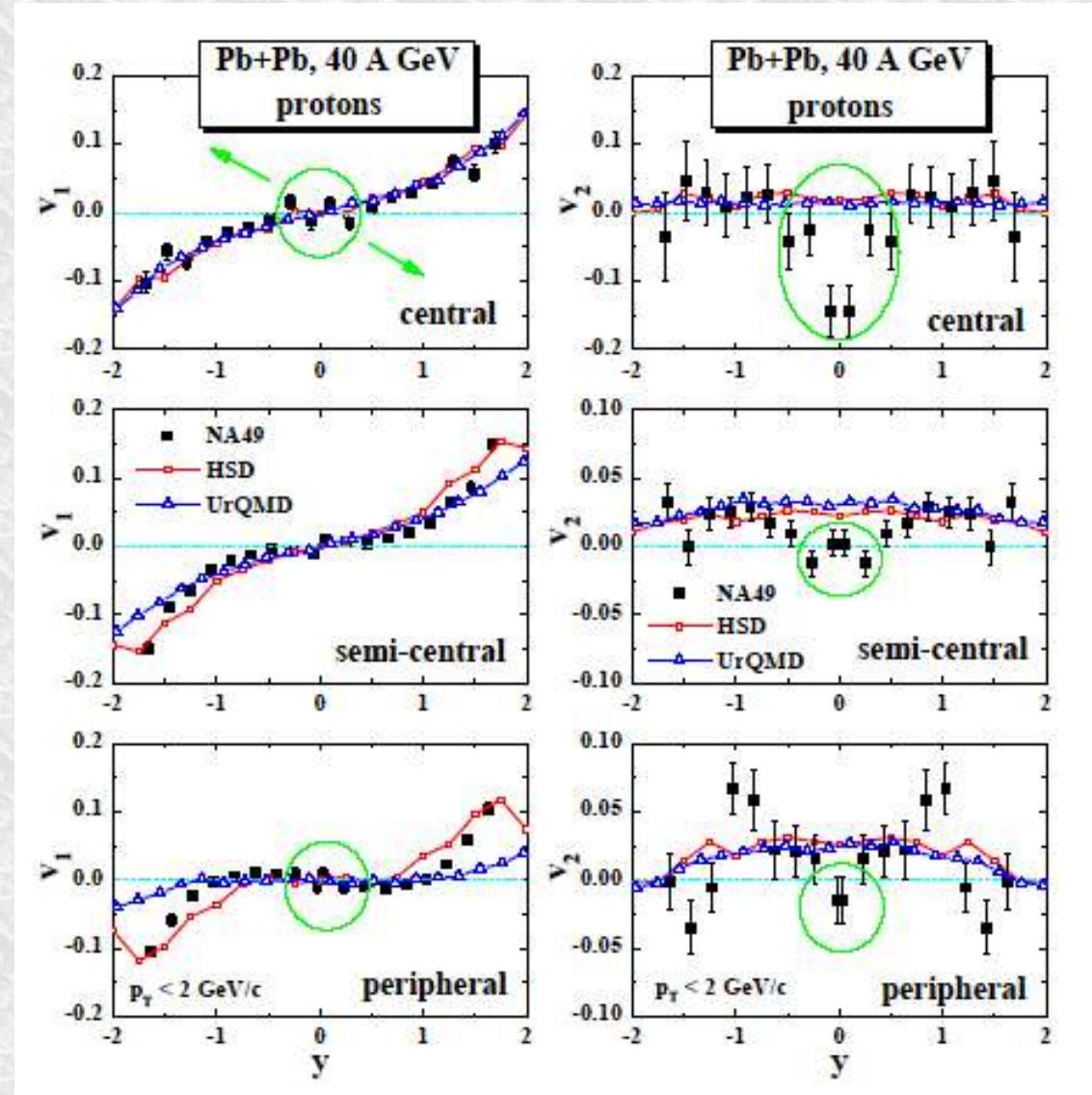
Mom. Dep. & Casc.



V2 is suppressed to around half

Dip of V2 at 40 A GeV: Phase Transition ?

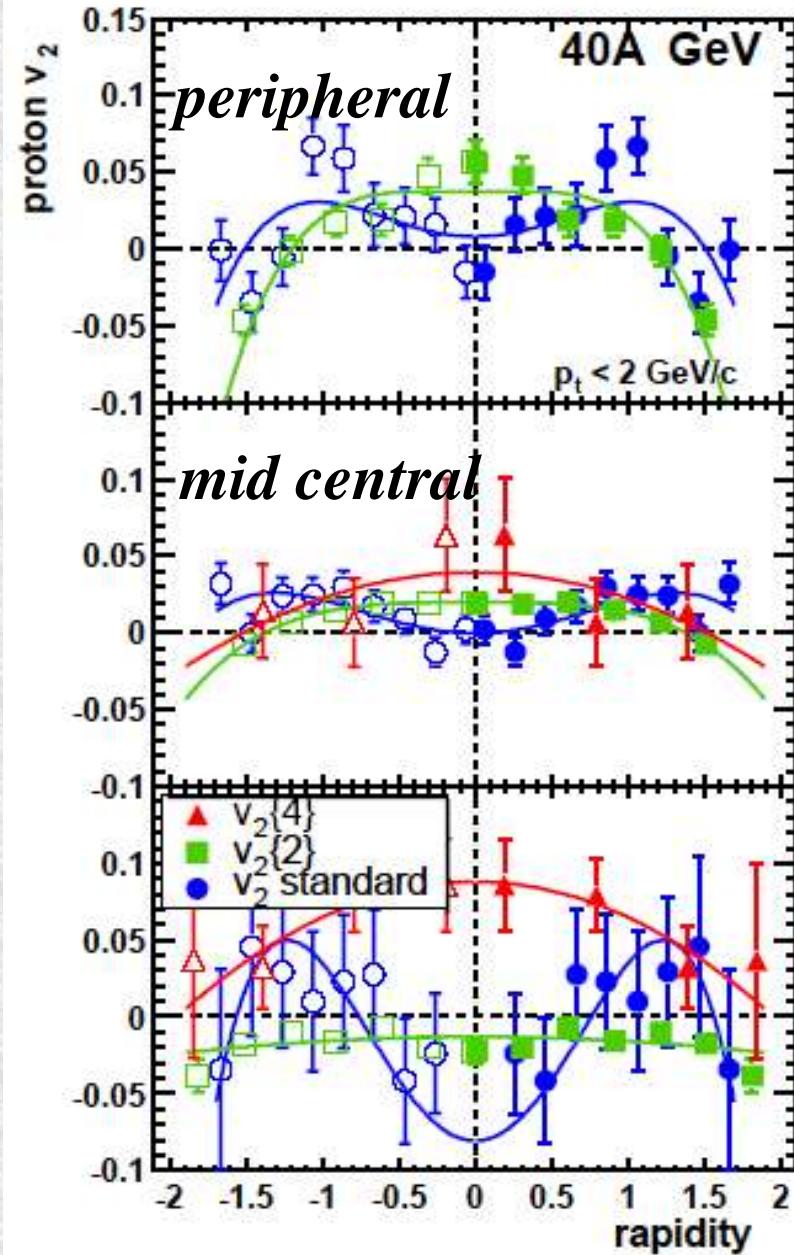
- Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.



(Stoecker et al., nucl-th/0412022)

Dip of V2 at 40 A GeV: Phase Transition ?

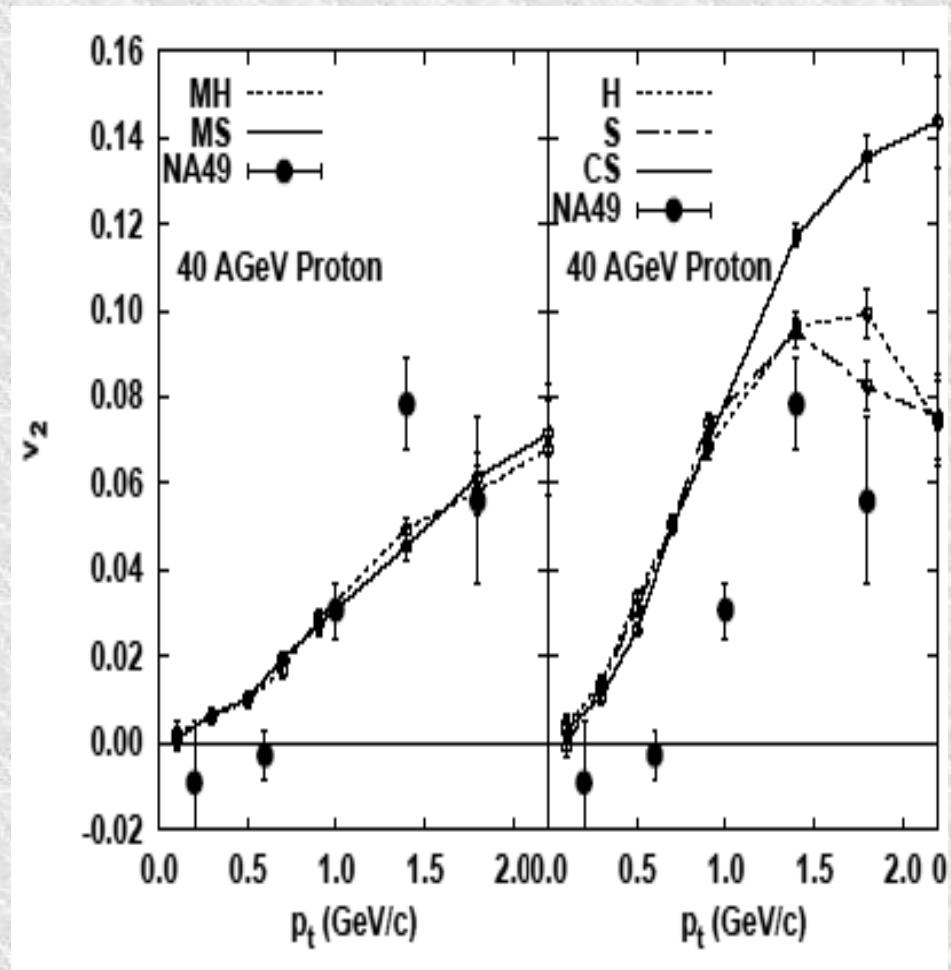
- Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
 - ★ We have to wait for better data.



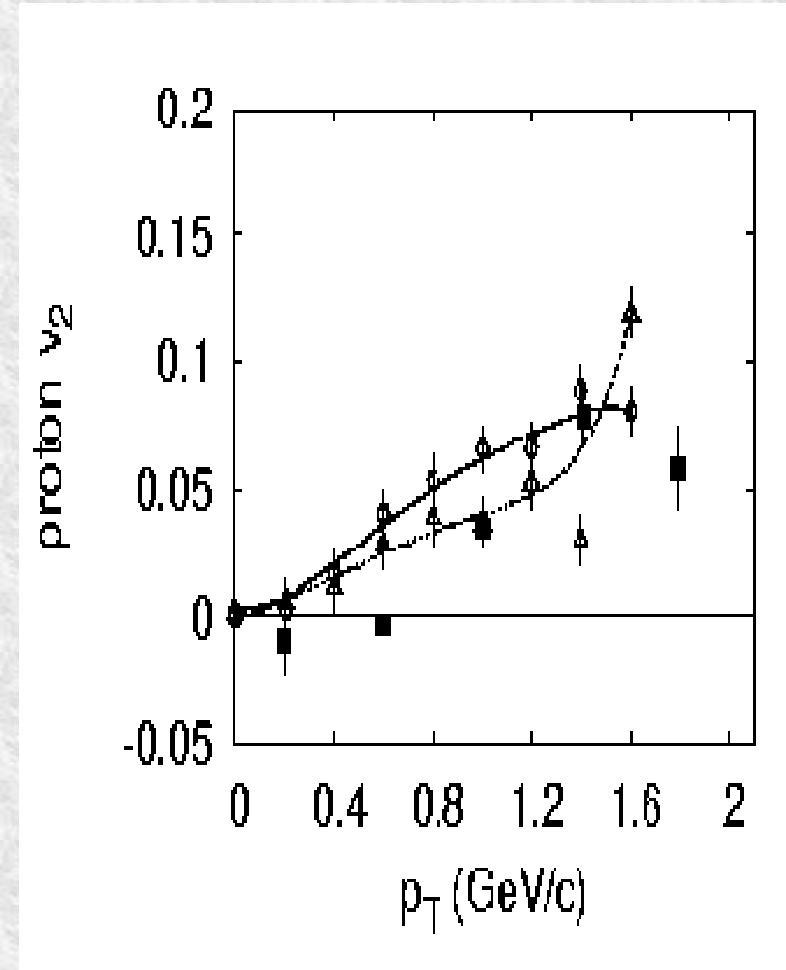
Elliptic Flow at SPS Energies (2)

(*Phys. Rev. C72, 064908 (2005); Phys. Rev. C77, 024904-1 (2008)*)

Mom. Dep. Casc. & Mom. Independ.

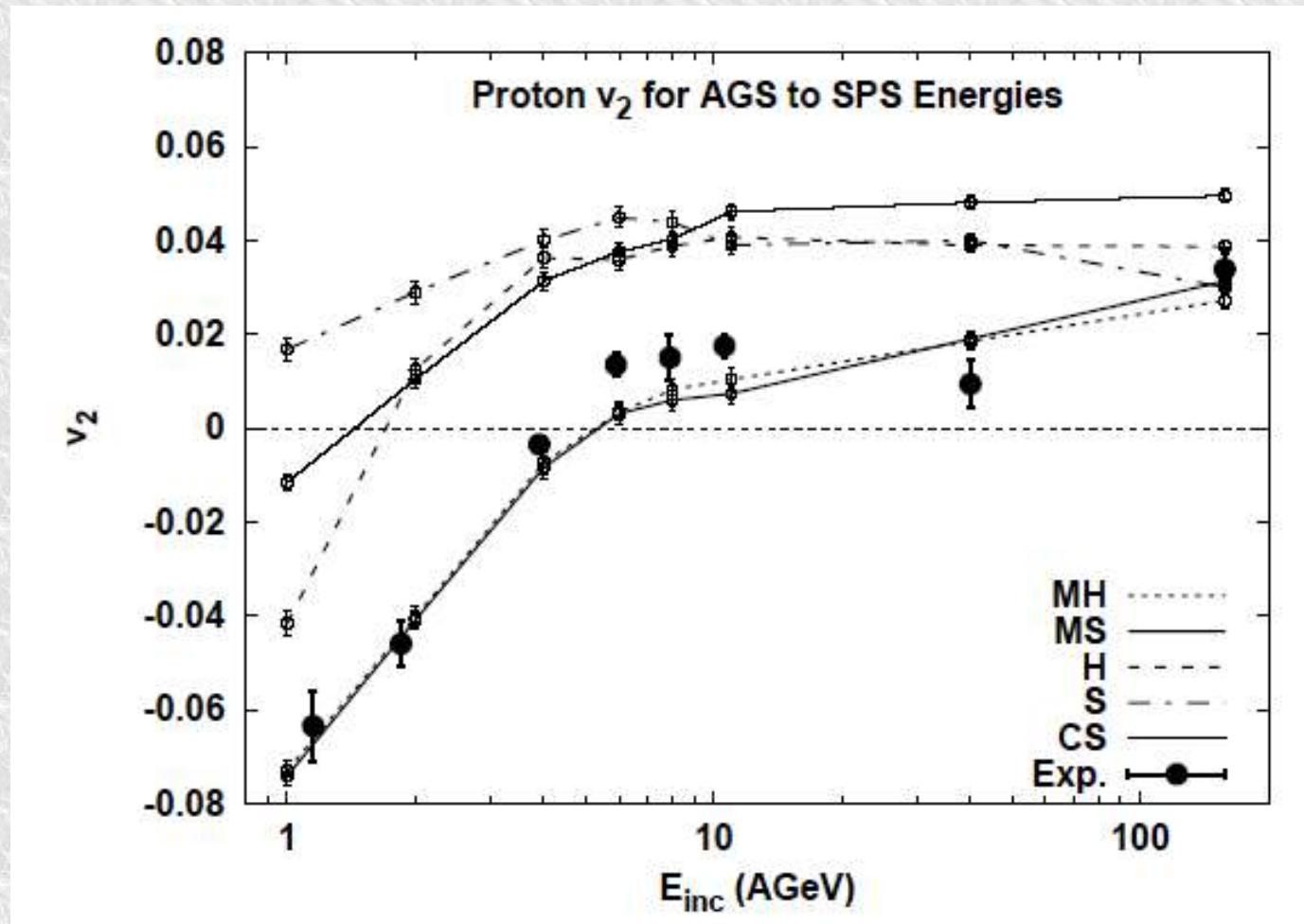


Mom. Dep.& Casc.



Incident Energy Dependence of Elliptic Flow

(Isse, Ohnishi, Otuka, Nara, Sahu, Phys. Rev. C72, 064908 (2005))



Globally Consistent Understanding of Flows at AGS and SPS energies is obtained !

Comparison with Other Models (I)

■ Boltzmann-Uehling-Uhlenbeck (BUU) Model

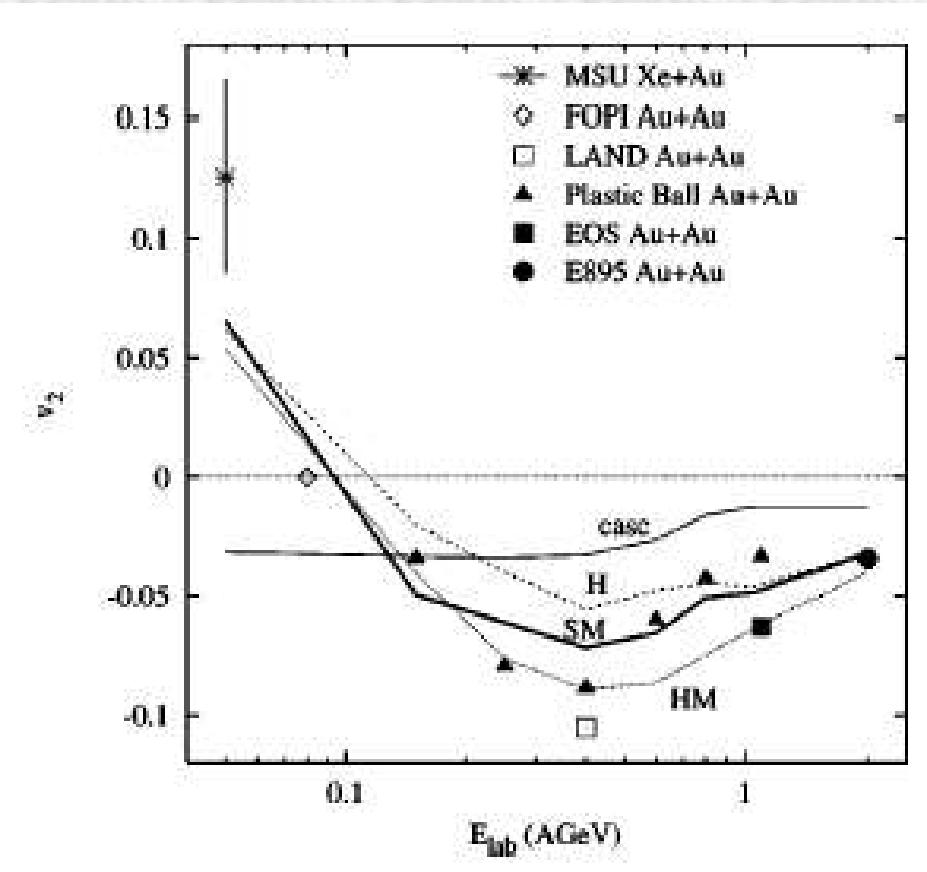
- ★ Lorentz Scalar Re-Interpretation of Non-Rel. MF

$$\begin{aligned}\varepsilon(\mathbf{p}, \rho) &= \sqrt{(m + U_s(\mathbf{p}, \rho))^2 + \mathbf{p}^2} \\ &= \sqrt{m^2 + \mathbf{p}^2} + U(\mathbf{p}, \rho).\end{aligned}$$

- ★ EOM: Suppressed Effects at High Energy

$$\dot{\mathbf{r}} = \frac{\partial H_{mf}}{\partial \mathbf{p}} = \frac{\mathbf{p}}{H_{mf}} + \frac{m_0 + s}{H_{mf}} \frac{\partial s}{\partial \mathbf{p}},$$

$$\dot{\mathbf{p}} = - \frac{\partial H_{mf}}{\partial \mathbf{r}} = - \frac{m_0 + s}{H_{mf}} \frac{\partial s}{\partial \mathbf{r}},$$



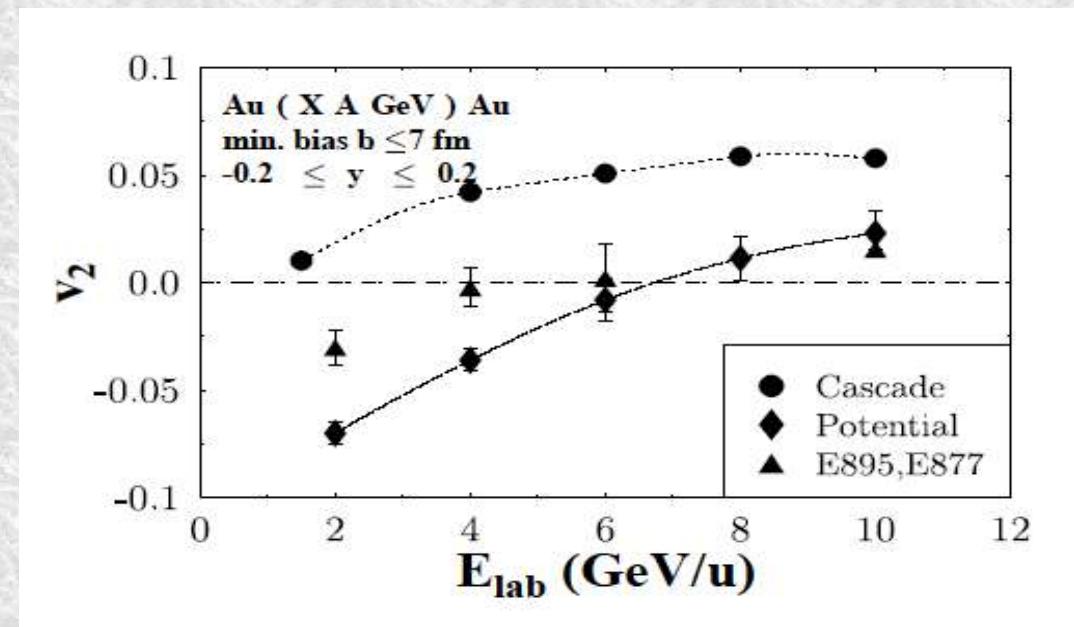
Mild Inc. Energy Deps.

(Larionov, Cassing, Greiner, Mosel, PRC62(2000),064611)

Comparison with Other Models (II)

■ UrQMD

- ★ *Strong Effects of “Hard” EOS*
- ★ *No Mom. Dep. MF is applied*
- ★ *No “Covariant Distance” is used in potential estimate.*



(Bass *et al.*, Prog. Part. Nucl. Phys. 41(1998),255)

Summary

- *Collective Flows is believed to be sensitive to EOS. Determination of EOS from Flows is an Old but Current (i.e. Long Standing) Problem.*
- *Momentum Dep. of Potential is Essential to understand Flows at High Energies, esp. above 1 A GeV.*
- *Hadronic Cascade with Momentum Dependent Mean Field works systematically from 2 AGeV to SPS (40 AGeV) energies.*
- *Precise estimate is still required to determine EOS. Especially, precise v1 and v2 measurements are very important to see “Baryon Rich QGP Formation” and EOS*
- *Let's wait for GSI future Project (CBM)!*

Collaborators:

- *U. Mosel (Giessen, Germany)*
- *W. Cassing (Giessen, Germany)*
- *Y. Nara (Akita, Japan)*
- *A. Ohnishi (Kyoto, Japan)*

Thank you!

