Multiplicity distributions: medium dependence in MLLA

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In collaboration with Néstor Armesto and Carlos Pajares

Introduction

standard QCD radiation pattern

extensively tested by jet measurements in high energy in e^+e^- and $pp(p\bar{p})$ collisions $\longrightarrow$ well described by MLLA

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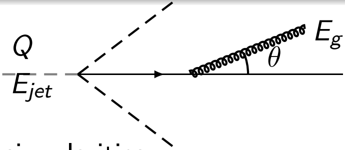
dense medium created $\longrightarrow$ distortion of the radiation pattern

We introduce the medium in the QCD equations in MLLA

to calculate the modification in the shape of the multiplicity distributions of the quark and gluon jets

Singularities: soft and collinear



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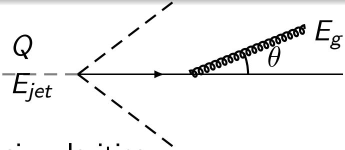

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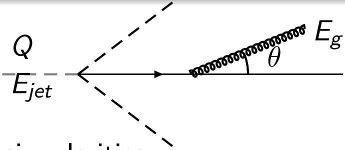

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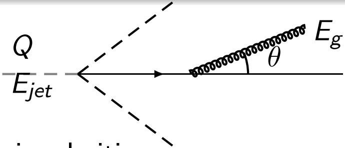

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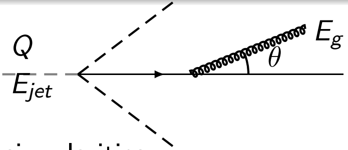

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Singularities: problems with convergence of the series

At small x soft and collinear logs have to be resummed

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  - SL corrections to DLA:

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Medium-induced gluon radiation

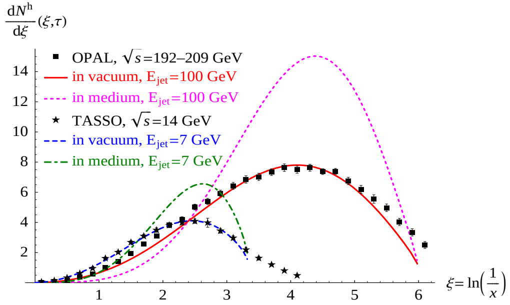
- Heavy Ion Collisions: dense medium created. Jet traveling through it.
  - medium-induced radiation $\longrightarrow$ distortion of standard (vacuum) QCD radiation pattern

- Most of the present phenomenology only for leading partons

- Study of the subleading structure is becoming experimentally accessible
  - RHIC
  - LHC

Medium-induced gluon radiation

- We use the simple prescription by Borghini and Wiedemann to study the effect of the medium-modification of the branching process
  - treats leading and subleading branchings on the same footing
  - ensures energy-momentum conservation at each splitting



$$\boxed{\xi = \ln\left(\frac{E_{jet}}{p}\right)}$$

$$\boxed{f_{med} = 0.8}$$

Introducing the medium

- We introduce a modification in the splitting functions
- Enhancement of the infrared parts of the kernels

$$K_G^G(x) = \frac{1+f_{med}}{x} - (1-x)[2 - x(1-x)] \qquad g \to gg$$

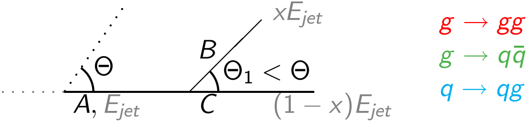
$$K_G^Q(x) = \frac{1}{4N_c}\left[x^2 + (1-x)^2\right] \qquad g \to q\bar{q}$$

$$K_Q^G(x) = \frac{C_F}{N_c}\left[\frac{1+f_{med}}{x} - 1 + \frac{x}{2}\right] \qquad q \to gq$$

Borghini and Wiedemann, 2005

N_c: the number of colors, $C_F = \frac{N_c^2-1}{2Nc} = 4/3$: Casimir factor

Evolution equations in MLLA



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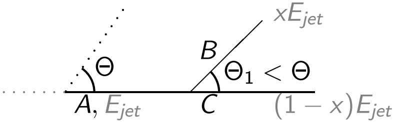

$g \longrightarrow gg$
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$$A = g: \quad G'_G(y) = \int_0^1 dx K_G^G \gamma_0^2 [G_G(y+\ln x) G_G(y+\ln(1-x)) - G_G(y)]$$

$$+n_f \int_0^1 dx K_G^Q \gamma_0^2 [G_Q(y+\ln x) G_Q(y+\ln(1-x)) - G_G(y)]$$

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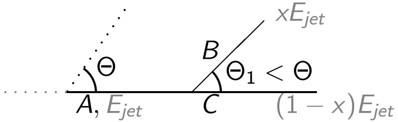

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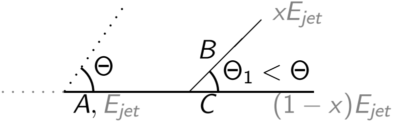

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n_f: number of active flavors, $\quad \gamma_0^2 = \frac{2N_c\alpha_s}{\pi}, \quad y = \ln \frac{E_{jet}\Theta}{Q_0}$

The equations

- Probability distributions in terms of the normalized factorial moments M_q

$$G_i(y,z) = \sum_{n=0}^{\infty}(z+1)^n P_n^i = \sum_{q=0}^{\infty} \frac{z^q}{q!}\langle n_i\rangle^q M_q, \qquad (i = G, Q)$$

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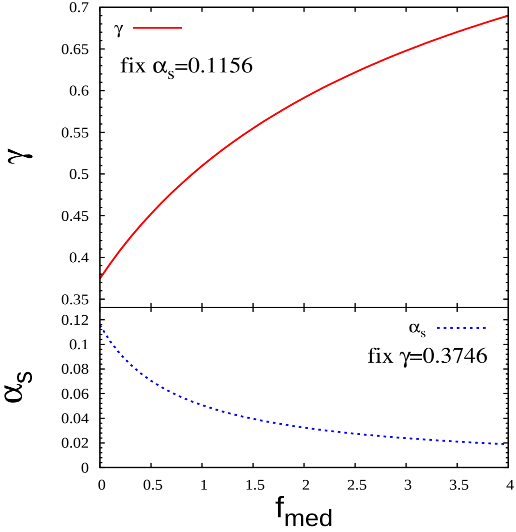
    - Solving $\mathcal{O}(z)$: $\gamma(f_{med})$ and $r(f_{med})$
- $\mathcal{O}(z^2)$ equations: known the parameters form $\mathcal{O}(z)$ we calculate $M_2(f_{med})$ for both α_s and γ fix, and thus the dispersion

First order equations I

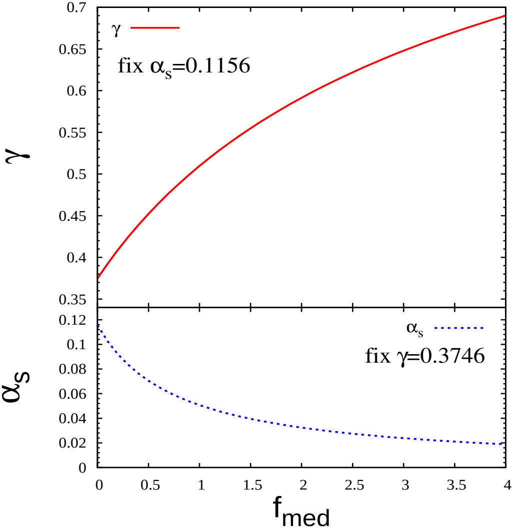


- Fix γ:

  $\boxed{\alpha_s(f_{med} \to \infty) \to 0}$

  physical sense?: if $f_{med} \propto \hat{q} \propto T^3$, we observe a drastic decrease in α_s for tiny increase in T

First order equations I



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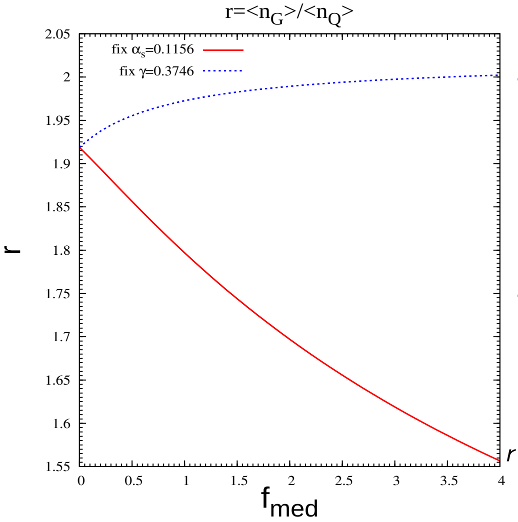
First order equations II



- Fix α_s:

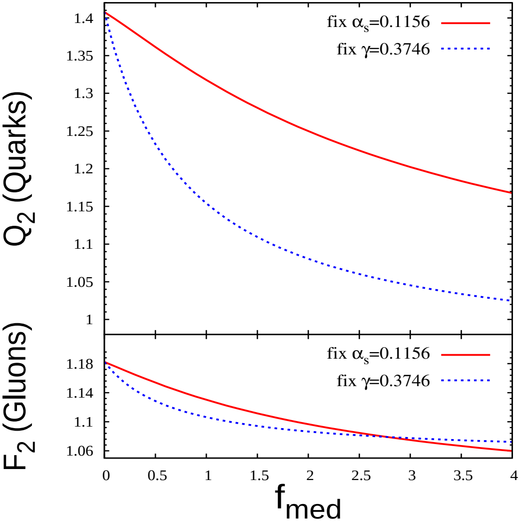
$$\boxed{r(f_{med} \to \infty) \to 1}$$

- Fix γ:

$$\boxed{r(f_{med} \to \infty) \simeq 2.02}$$

$$r \xrightarrow[f_{med}\to\infty]{} \gamma \left[[\gamma_E + \Psi(\gamma)] \left(1 - \frac{N_c}{C_f}\right) + \frac{1}{\gamma} \right]$$

Second order normalized factorial moments



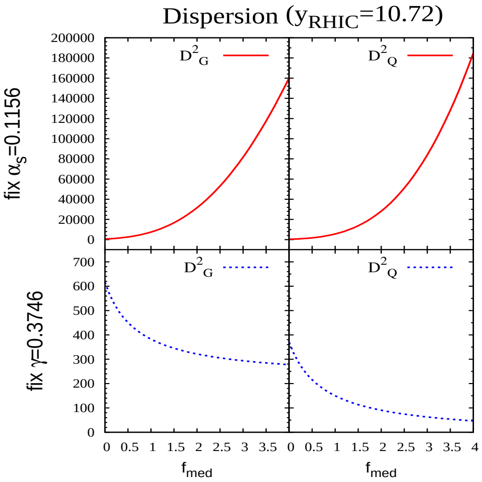
$$\boxed{M_2 = \frac{\langle n_i(n_i-1)\rangle}{\langle n_i\rangle^2}}$$

$$\boxed{F_2 = \frac{\langle n_G(n_G-1)\rangle}{\langle n_G\rangle^2}}$$

$$\boxed{Q_2 = \frac{\langle n_Q(n_Q-1)\rangle}{\langle n_Q\rangle^2}}$$

- Decrease of M_2 with the contribution of the medium

Dispersion: $D_i^2 = \langle n_i^2 \rangle - \langle n_i \rangle^2$



$$\boxed{D_i^2 = \langle n_i \rangle^2 (M_2 - 1 + \langle n_i \rangle^{-1})}$$

$$\boxed{D_G^2 = e^{2\gamma y}(F_2 - 1 + e^{-\gamma y})}$$

$$\boxed{D_Q^2 = r^{-1} e^{2\gamma y}(Q_2 - 1 + r e^{-\gamma y})}$$

Fix α_s

increase in dispersion (expected)

Fix γ

drastic decrease in dispersion

Conclusions and Outlook

- We have introduced the medium as a multiplicative constant f_{med} in the singular parts of the splitting functions
- We have analyzed two cases:
  - We find an increase in the dispersion of the distribution when α_s is **fixed**; in disagreement with the γ **fix** case.
- **Next step**: introduce the properties of the medium, such as medium length L and $\hat{q}$, and also a dependence with the jet energy
- See the modification of the multiplicity distribution with those properties

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  - We find an increase in the dispersion of the distribution when α_s is **fixed**; in disagreement with the γ **fix** case.
- **Next step**: introduce the properties of the medium, such as medium length L and $\hat{q}$, and also a dependence with the jet energy
- See the modification of the multiplicity distribution with those properties

Work in progress

Thank you!