Transport Coefficients in semi-QGP

Hot quarks 2008 August 19th

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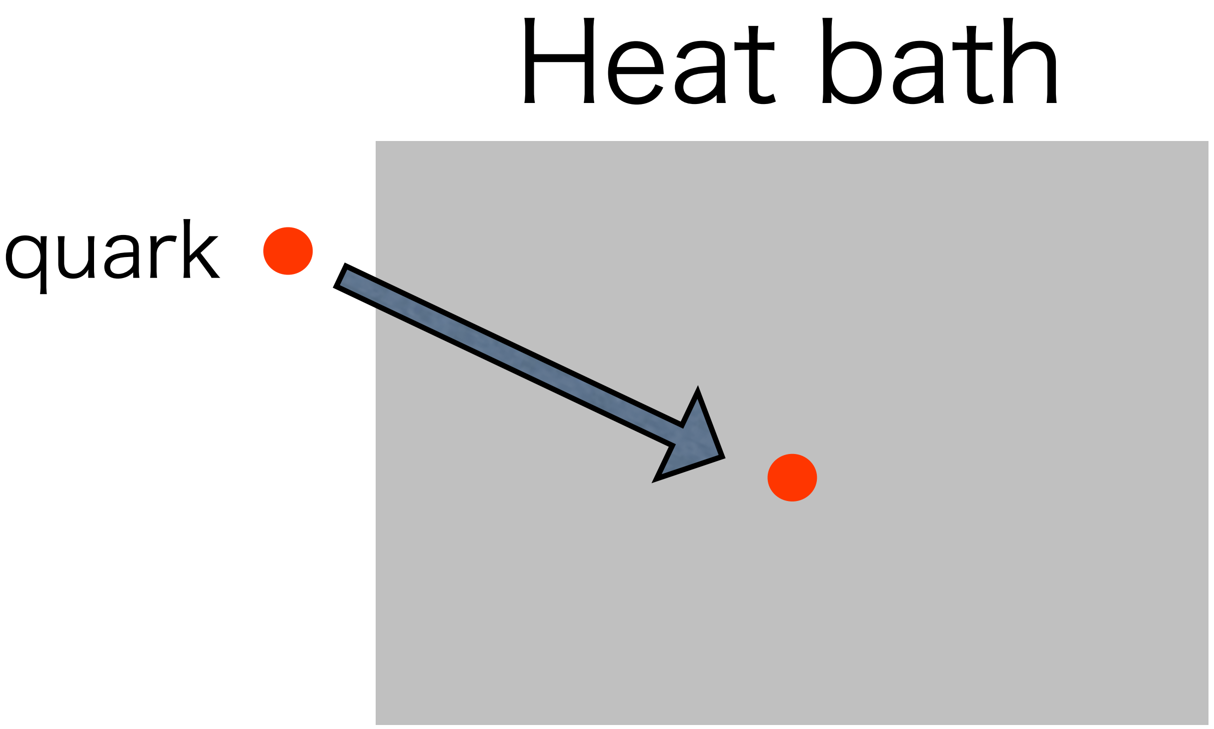
Collaboration with R. D. Pisarski (BNL)

Based on arXiv:0803.0453[hep-ph]

What is "Semi"-QGP?



$$\langle \mathrm{tr} L_r \rangle \sim \exp\left(-f_r/T + \mathrm{pert.}\right)$$

one particle free energy

$$f_r = \begin{cases} 0 & \text{(complete-QGP)} \\ \text{finite} & \text{(semi-QGP)} \\ \infty & \text{(confined)} \end{cases}$$

fr depends upon the color representation, like chemical potential.

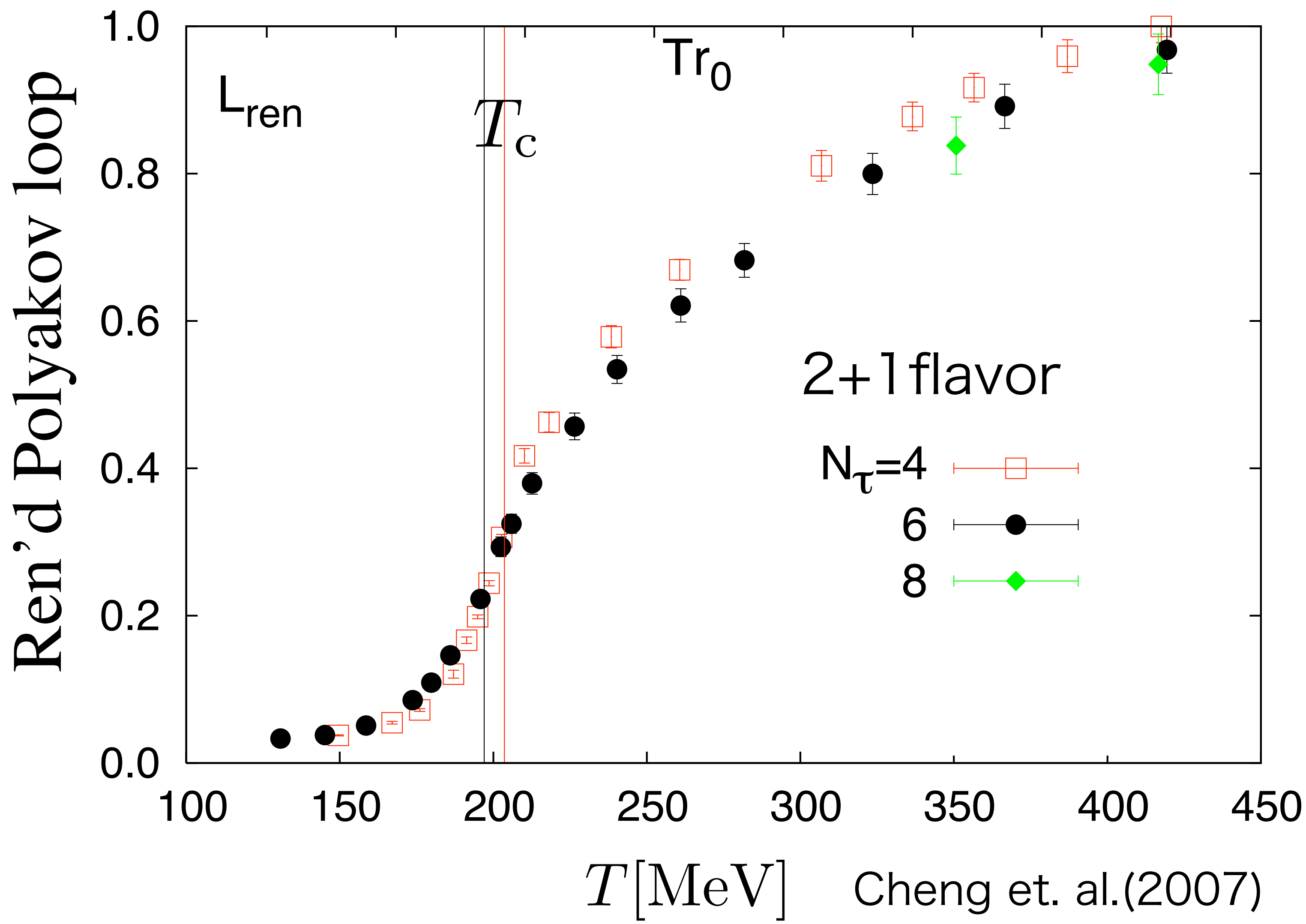
Semi-QGP = partially deconfined QGP.

Semi-QGP is qualitatively different from the complete QGP.

Semi-QGP Window

Hadronic | Semi-QGP | Complete-QGP

$\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle \approx 0$ | $\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle < 1$ | $\left\langle \frac{1}{N_c}\mathrm{tr}L \right\rangle \simeq 1$



Semi-QGP Window

$$0.8T_c - 2T_c$$

Maybe RHIC probes the semi-QGP!!

Pressure, susceptibilities change dramatically in the semi-QGP.
How about transport coefficients?

Previous Work

shear viscosity

bulk viscosity

★ Perturbation theory

$$\alpha_s \ll 1$$

$$\eta \sim \frac{T^3}{\alpha_s^2 \log[1/\alpha_s]}$$

Hosoya, Sakagami, Takao ('84);Hosoya, Kajantie ('85);Baym, Monien, Pethick, Ravenhall ('90), ('91); Arnold, Moore, Yaffe ('00), ('03)

$$\zeta \sim \frac{\alpha_s^2 T^3}{\log[1/\alpha_s]}$$

Arnold, Dogan, Moore ('06)

★ $\mathcal{N} = 4$ Super Yang-Mills

$N_c \to \infty$

$\alpha_s N_c \to \infty$

Policastro, Son, Starinets ('01)
Kovtun, Son, Starinets ('04)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

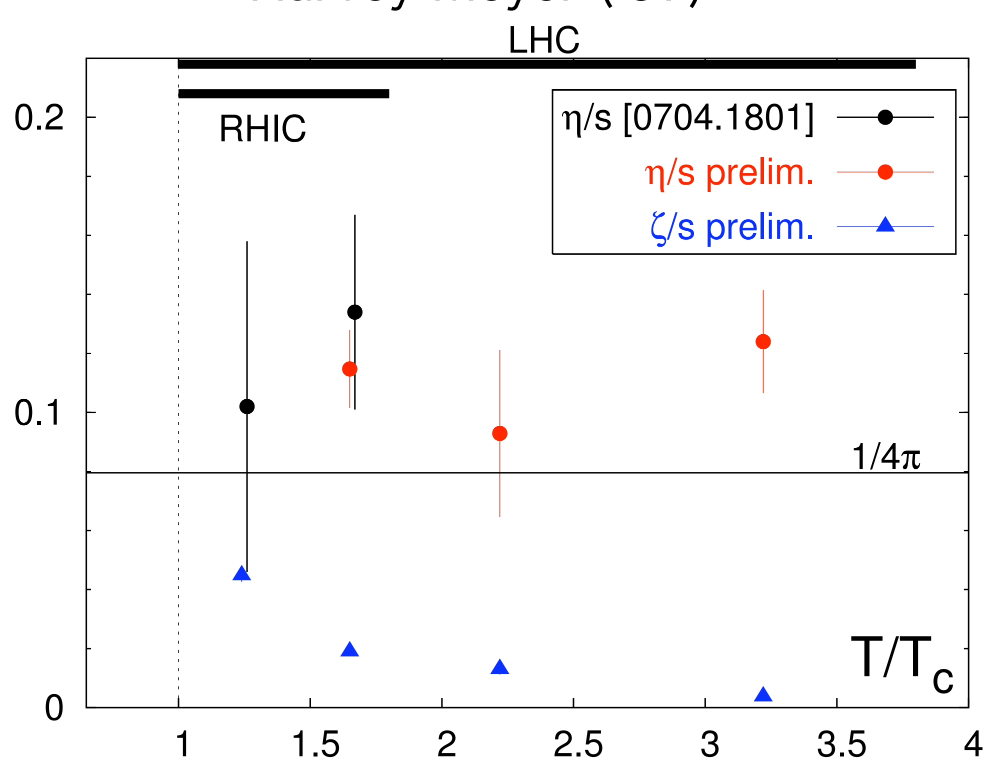
$$\frac{\zeta}{s} = 0 \quad \text{conformal}$$

s: entropy density

★ Lattice

Karsch and Wyld('87); Sakai and Nakamura('04); Aarts, Allton, Justin Foley, Hands, Kim('07) Meyer ('07)

Harvey Meyer ('07)



Bulk viscosity near Tc

Kharzeev, Tuchin (07); Karsch, Kharzeev, Tuchin('07)

Hubner, Karsch, Pica ('08)

AnomalousViscosity

Asakawa, Bass, Muller('06)

Hadron phase

Gavin('85) Prakash, Prakash, Venugopalan, Welke('93)

Davesne('96) Dobado, Santalla('01)

Dobado, Llanes-Estrada('03) Chen, Nakano ('06)

Andez-Fraile, Nicola('06)

Itakura, Morimatsu, Otomo ('07)

Effective theory of Semi-QGP

In semi-QGP: $0 < \langle \frac{1}{N_{\mathrm{c}}} \mathrm{trL} \rangle < 1$ $\quad \mathrm{trL} = \mathrm{trP}\mathrm{e}^{ig\int d\tau A_0} = \sum_a \mathrm{e}^{i\theta(a)}$

A_0: classical background with eigenvalues, $T\theta/g$

+ fluctuation

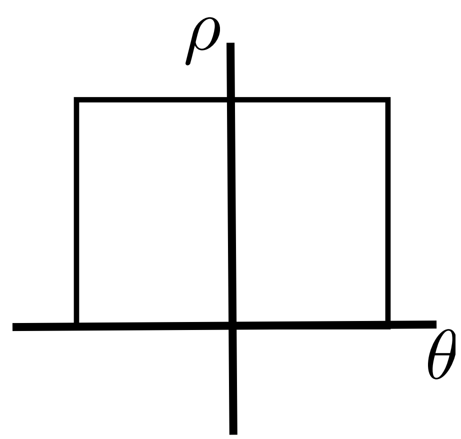
Color sum $\xrightarrow{N_{\mathrm{c}} \to \infty}$ average of eigenvalue with spectral density, $\rho(\theta)$

$$\langle \frac{1}{N_{\mathrm{c}}} \sum_{\mathrm{color}} \mathcal{O} \rangle = \int d\theta \rho(\theta) \mathcal{O}(\theta)$$

e.g. Polyakov loop $\langle \frac{1}{N_{\mathrm{c}}} \mathrm{tr} L \rangle = \int d\theta \rho(\theta) \mathrm{e}^{i\theta}$

Spectral density

★ Confined phase

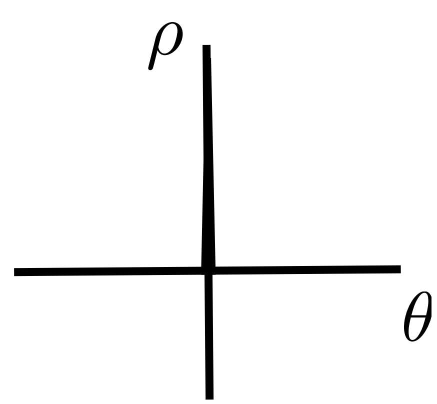


Constant dist., $\rho(\theta) = \dfrac{1}{2\pi}$

All Polyakov loops vanish

$$\frac{1}{N_{\rm c}}{\rm trL}^n = \int d\theta \rho(\theta) {\rm e}^{in\theta} = 0$$

★ Complete QGP

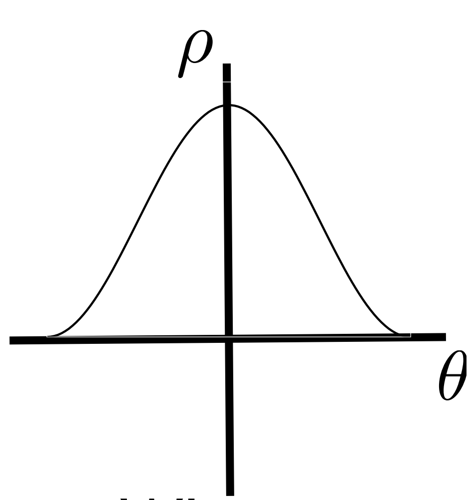


Delta function dist., $\rho(\theta) = \delta(\theta)$

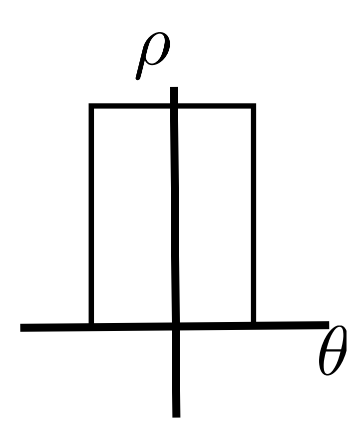
All Polyakov loops unity

$$\frac{1}{N_{\rm c}}{\rm trL}^n = 1$$

★ Semi-QGP



Gross-Witten model



Step function

$$0 < \frac{1}{N_{\rm c}}{\rm trL}^n < 1$$

Use two eigenvalue dist.'s.

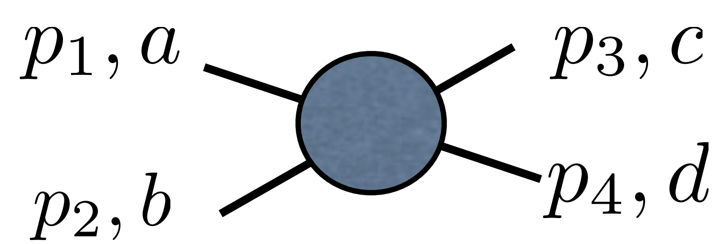
Kinetic Theory

Boltzmann Equation

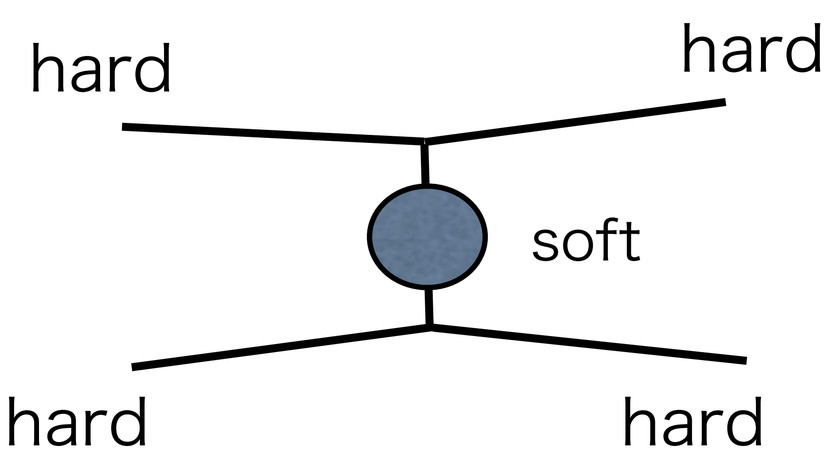
$$\frac{\partial}{\partial t} f^a + \boldsymbol{v}_p \cdot \frac{\partial}{\partial \boldsymbol{x}} f^a + \boldsymbol{F}_{\text{ext}} \cdot \frac{\partial}{\partial \boldsymbol{p}} f^a = -\frac{1}{2} \sum_{\text{color,spin,flavor}} \int d\Pi |\mathcal{M}|^2 f^a f^b (1 \pm f^c)(1 \pm f^d)$$

Collision term

Two body scattering: $\mathcal{M} =$



Work only to leading log order. Only t-channel contributes.



soft gluon exchange

$$|\mathcal{M}|^2 \sim \frac{1}{(q^2 + m_{\text{D}}^2)^2}$$ (gluon exchange)

m_{D} :color dependent Debye mass

Solving Boltzmann eq. → **Shear viscosity**

Arnold, Moore, Yaffe ('00)

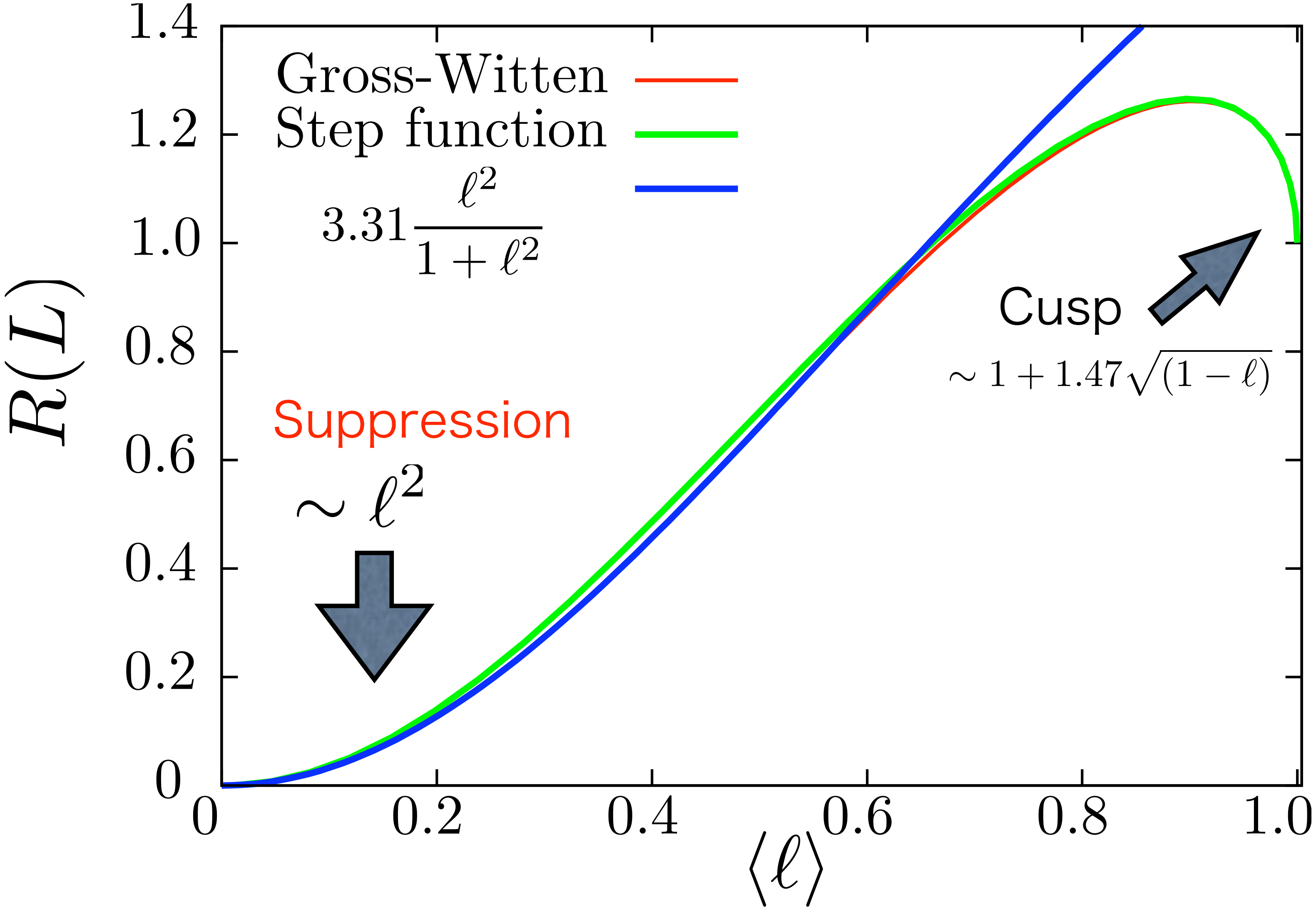
Viscosity in the semi-QGP: pure glue

Y.H., Pisarski('08)

Viscosity $\eta = \frac{cT^3}{g^4 \log(1/g)} R(\mathrm{L})$

perturbative modification

Normalization $R(\mathrm{L}=1)=1$



★ Semi-QGP

$\eta \sim |\ell|^2 T/\sigma$

σ :cross section

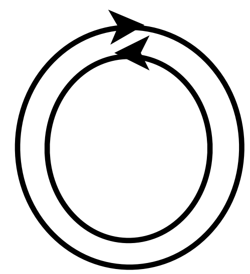
Unlike classical dilute gas

$\eta \sim T/\sigma$

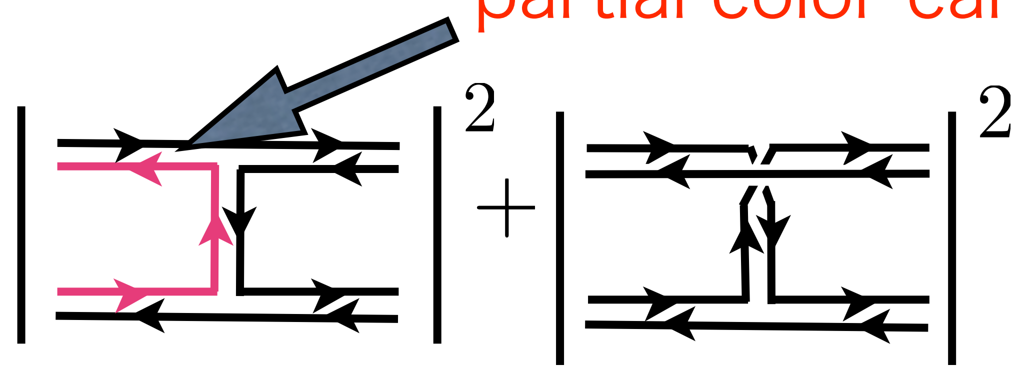
Suppression at small ℓ ! $R(\mathrm{L}) \sim \ell^2$

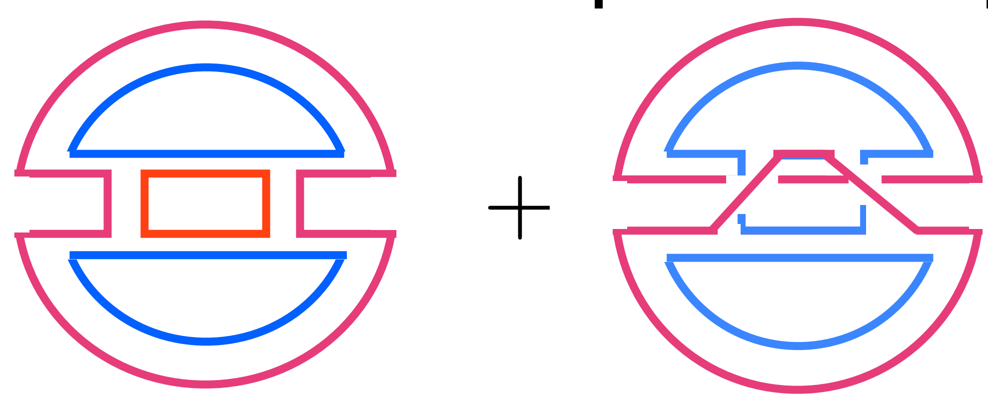
Little change between two eig. dist.'s.

Viscosity in the semi-QGP: pure glue(contd.)

$Df = S \sim$  $\sim N_c^2 \ell^2$

Collision term in Large Nc

$C \sim$ 

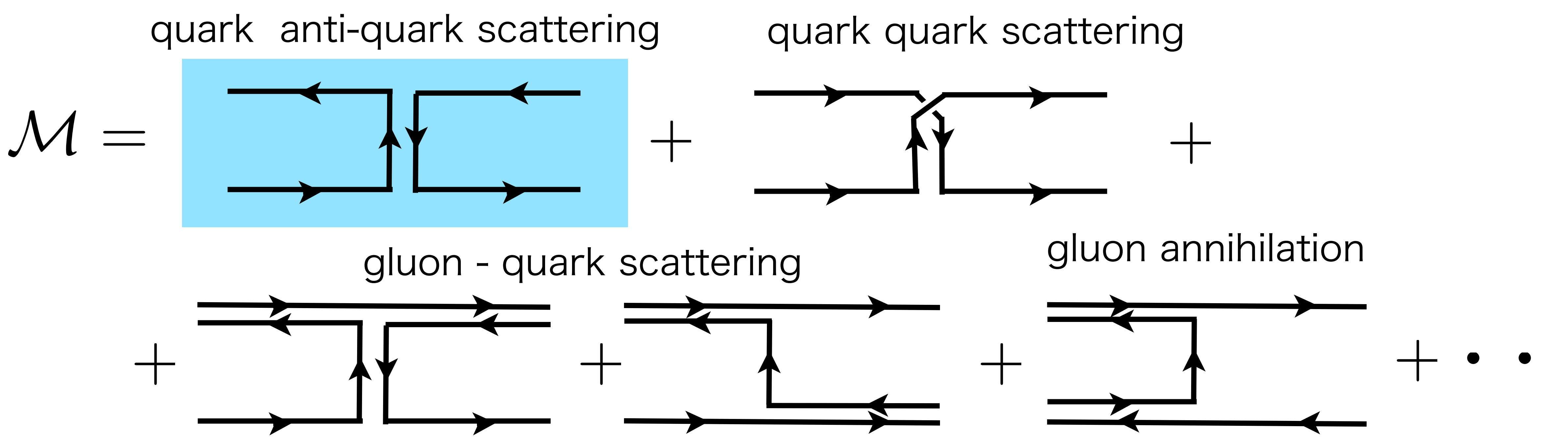
$=$  $\sim N_c^4(\ell^2 + \ell^4)$

at small ℓ

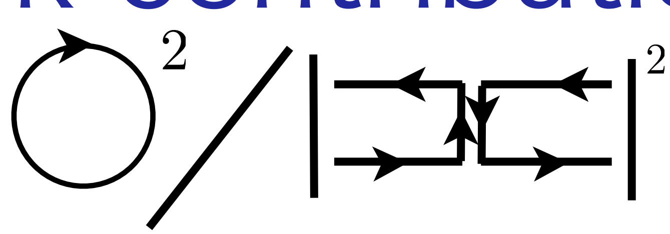
From Boltzmann Eq. $\eta = \frac{1}{15} S^t C^{-1} S \sim \frac{(\ell^2)^2}{\ell^2 + \ell^4} = \frac{\ell^2}{1+\ell^2}$

Viscosity in semi-QGP with quarks

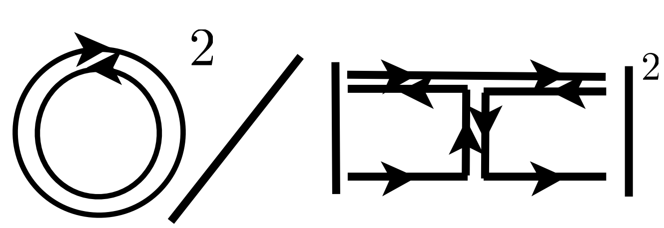
With quarks, more scattering channels.
Assume $N_{\rm f} \sim N_{\rm c} \gg 1$.



Quark contribution dominates.

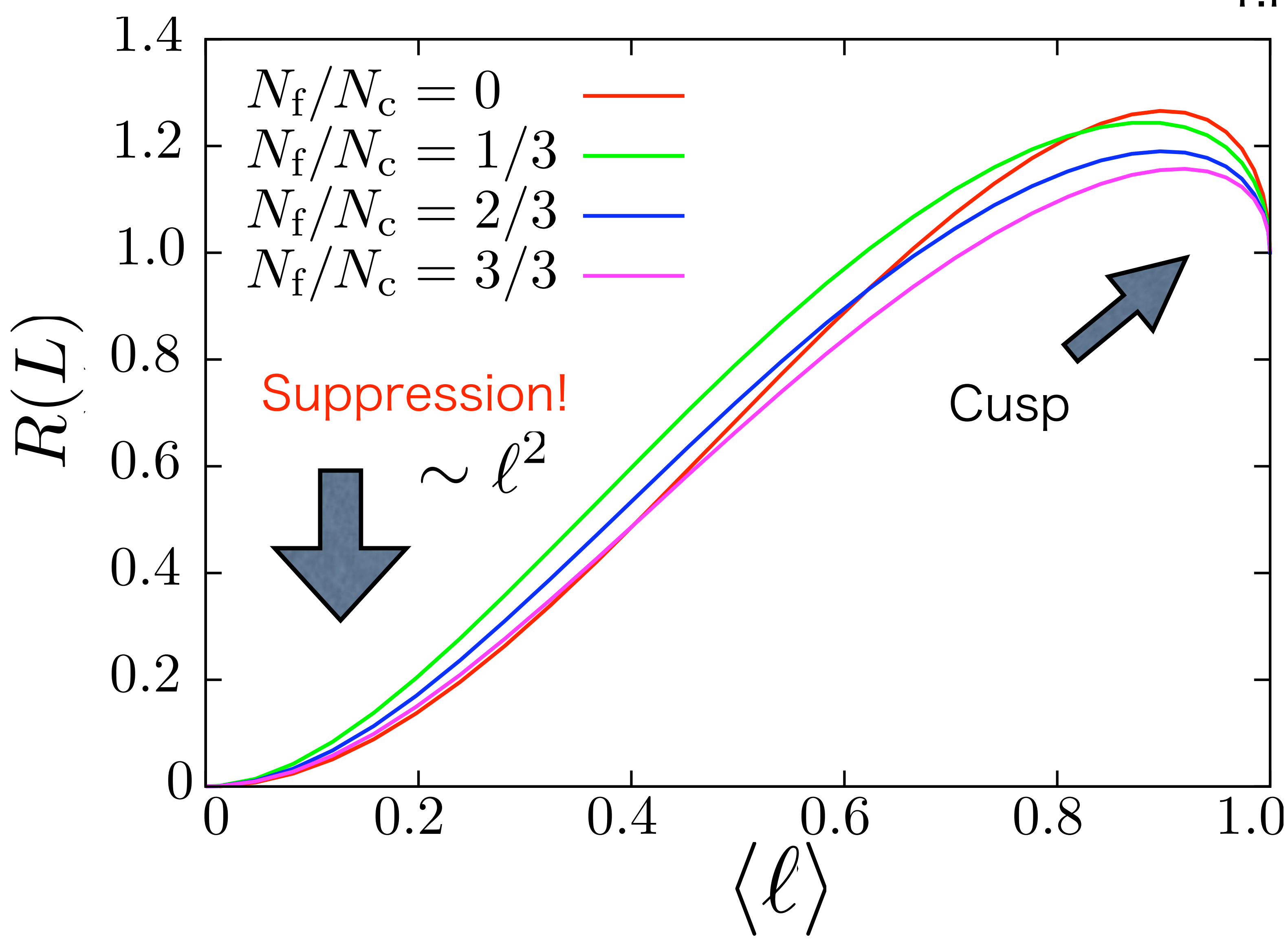
 $\sim \ell^2/1 = \ell^2$

Gluon contribution is suppressed,

 $\sim \ell^3$

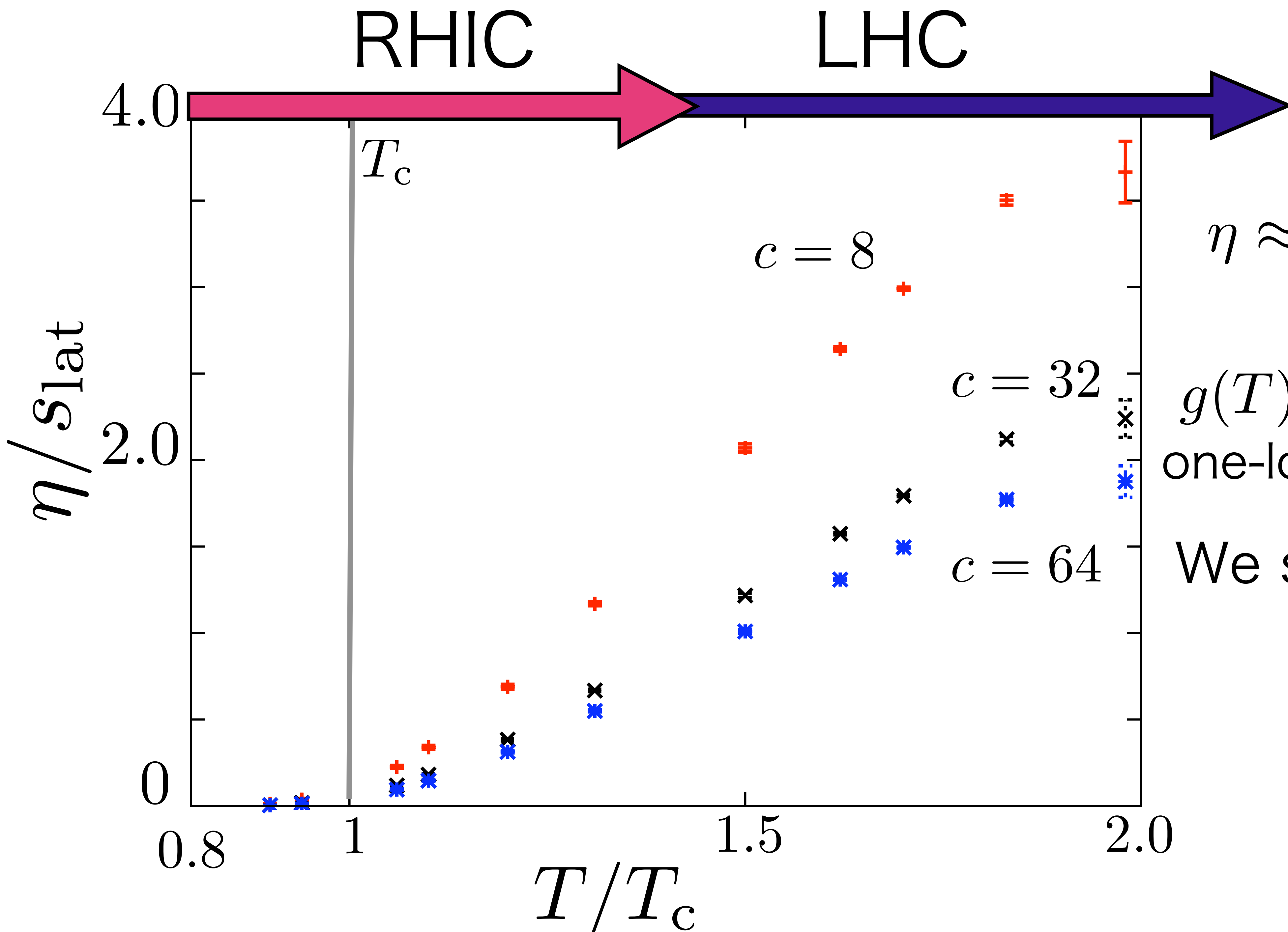
Viscosity in semi-QGP with quarks(contd.)

Y.H., Pisarski('08)



Quark contribution dominates.

Viscosity/Entropy



$$\eta \approx \frac{99.0 \times R(L)}{g^4(T)\ln(c/g(T))}$$

$g(T)$
one-loop running coupling

We set $\frac{g^2(T_\mathrm{c})}{4\pi} = \frac{1}{3}$

Lattice data from Cheng, et.al. PRD77, 014511(2008)

Summary

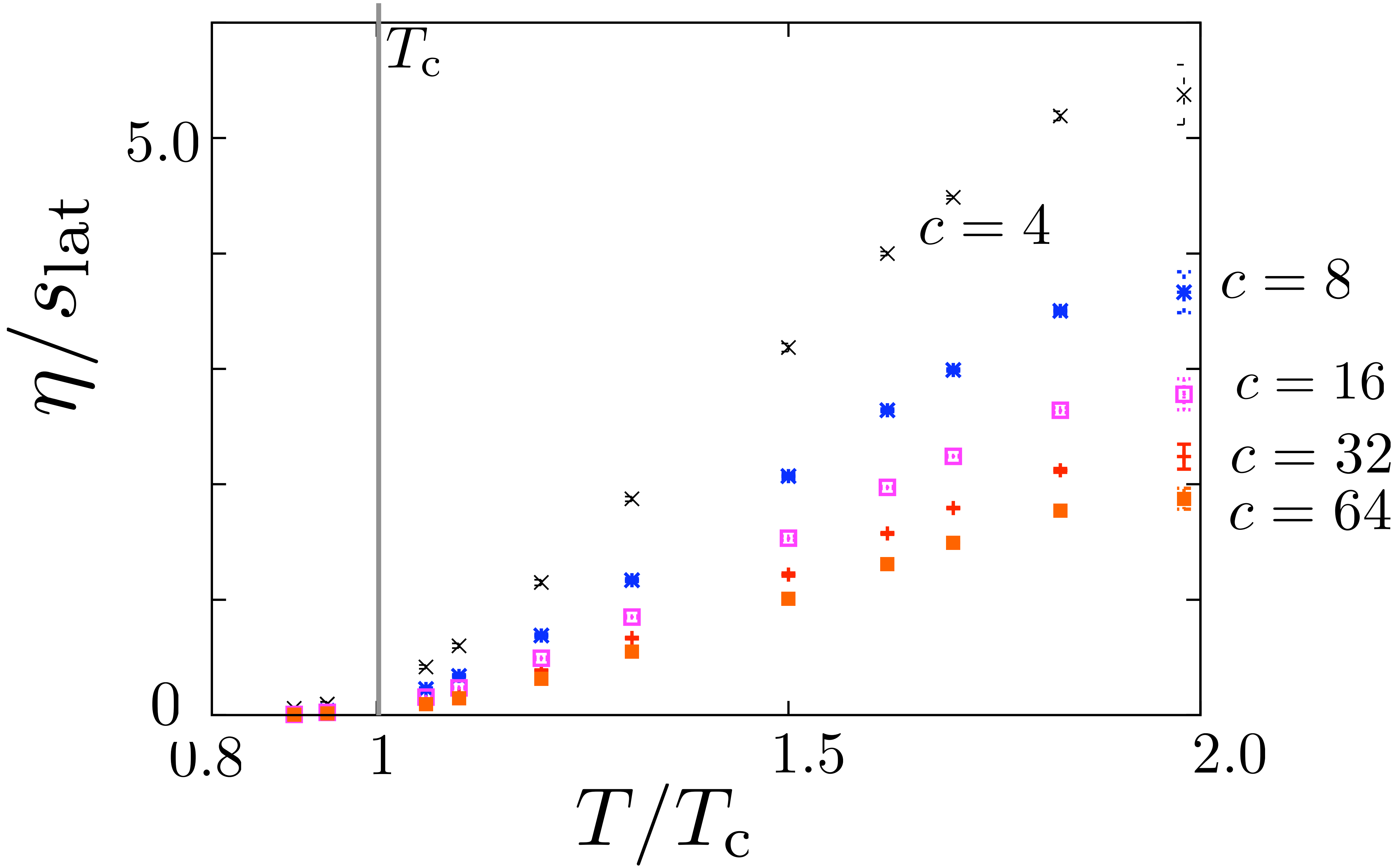
- Shear viscosity suppressed, near Tc, $\sim \ell^2$. Quarks dominates.

- RHIC - probes semi-QGP? If so, not only η, but R_{AA}, real photons, dileptons, also suppressed by powers of ℓ .

- LHC - into complete QGP?
If so, LHC $\neq$ RHIC, a BIG shear viscosity at LHC at short times.

Back up

Viscosity/Entropy



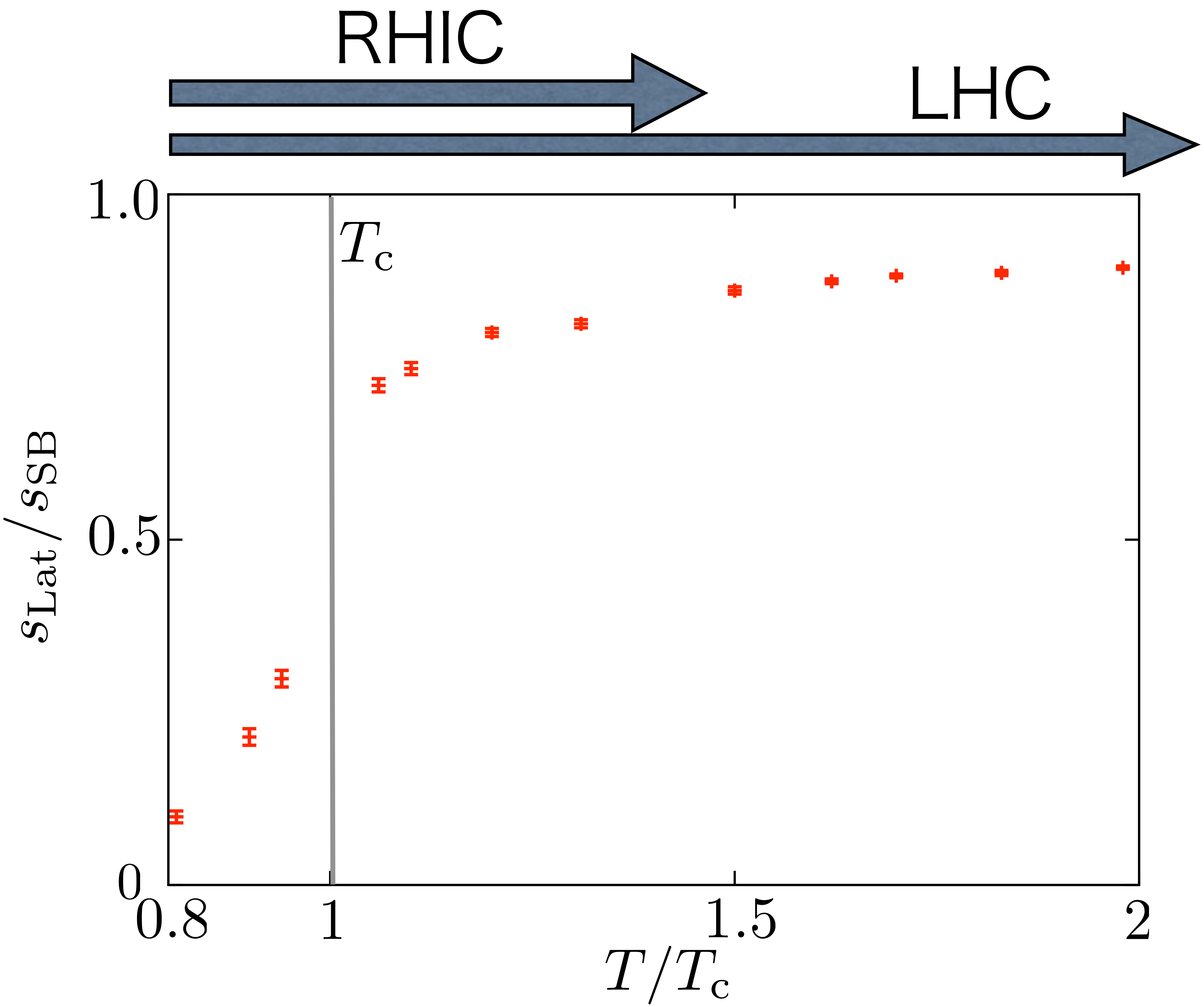
Lattice data from Cheng, et.al. PRD77,014511(2008)

one lop running coupling constant

$$g^2(k) = \frac{g^2}{1 + \frac{g^2}{(4\pi)^2}(\frac{11}{3}N_{\rm c} - \frac{2}{3}N_{\rm f})\log(k^2/M^2)}$$

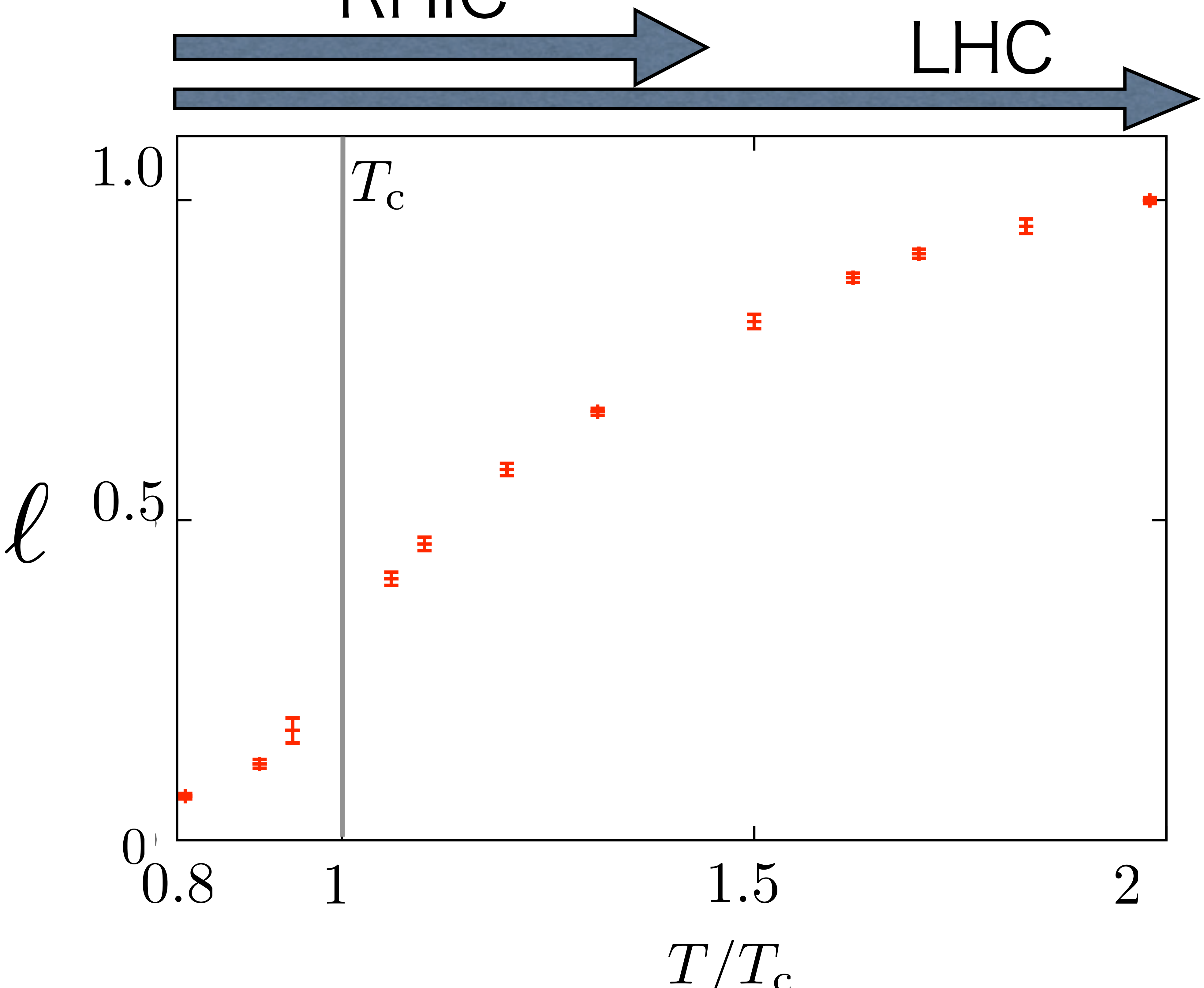
Nc=Nf=3

$$\alpha_{\rm s}(k) = \frac{\alpha_{\rm s}}{1 + 9\alpha_{\rm s}\log(k^2/M^2)}$$

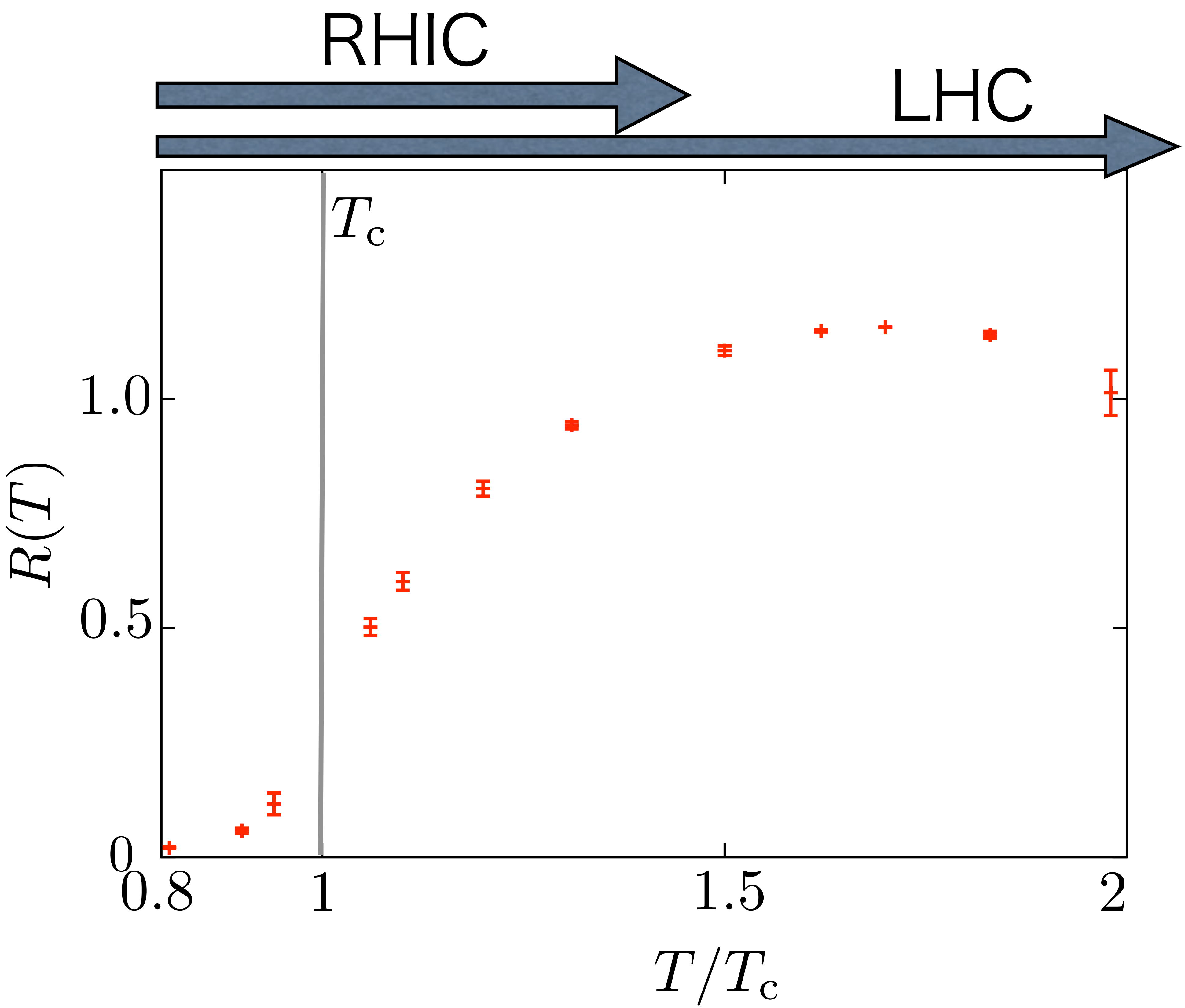


Lattice data from Cheng, et.al. PRD77,014511(2008)

Renormalized Polyakov loop

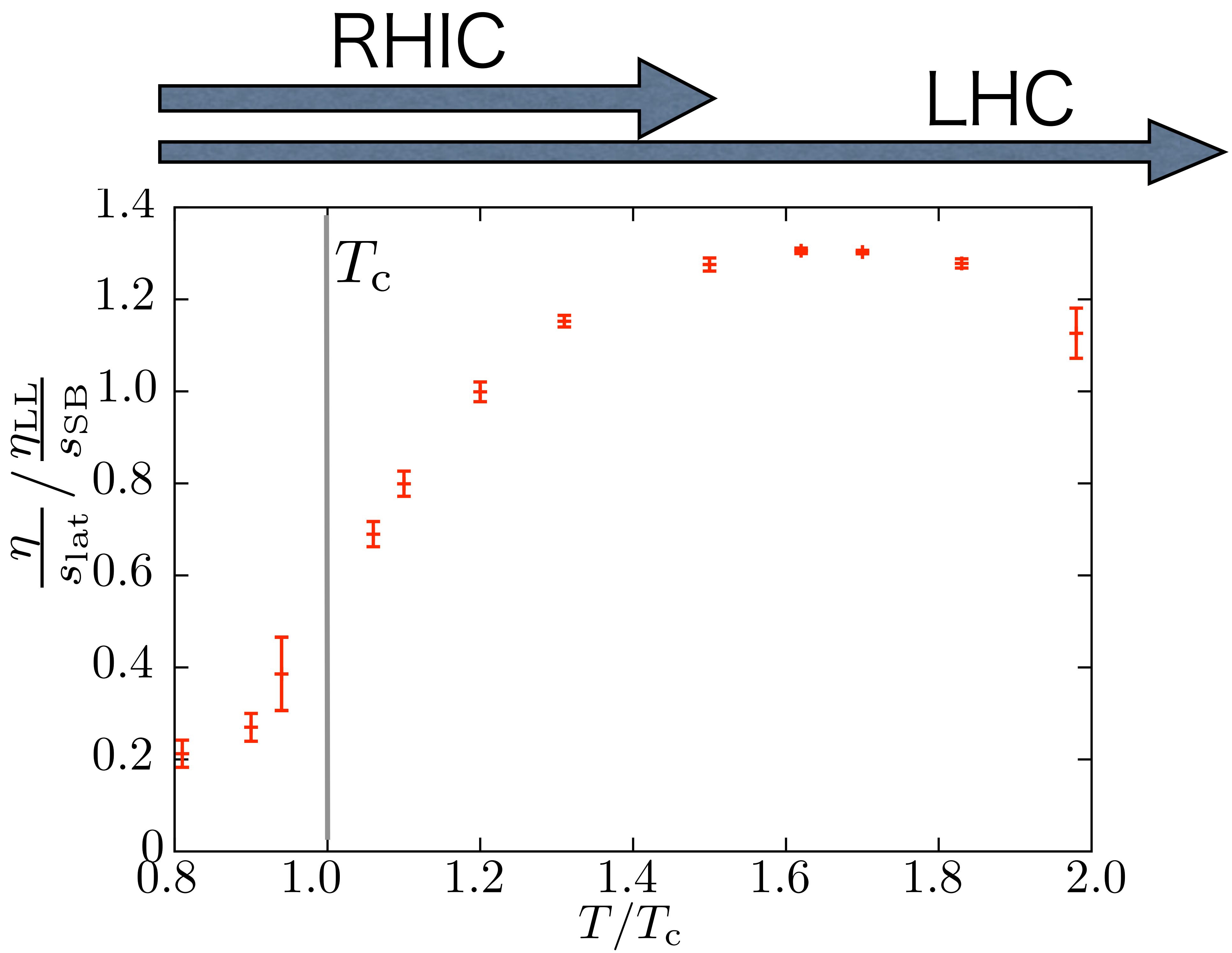


Lattice data from Cheng, et.al. PRD77,014511(2008)



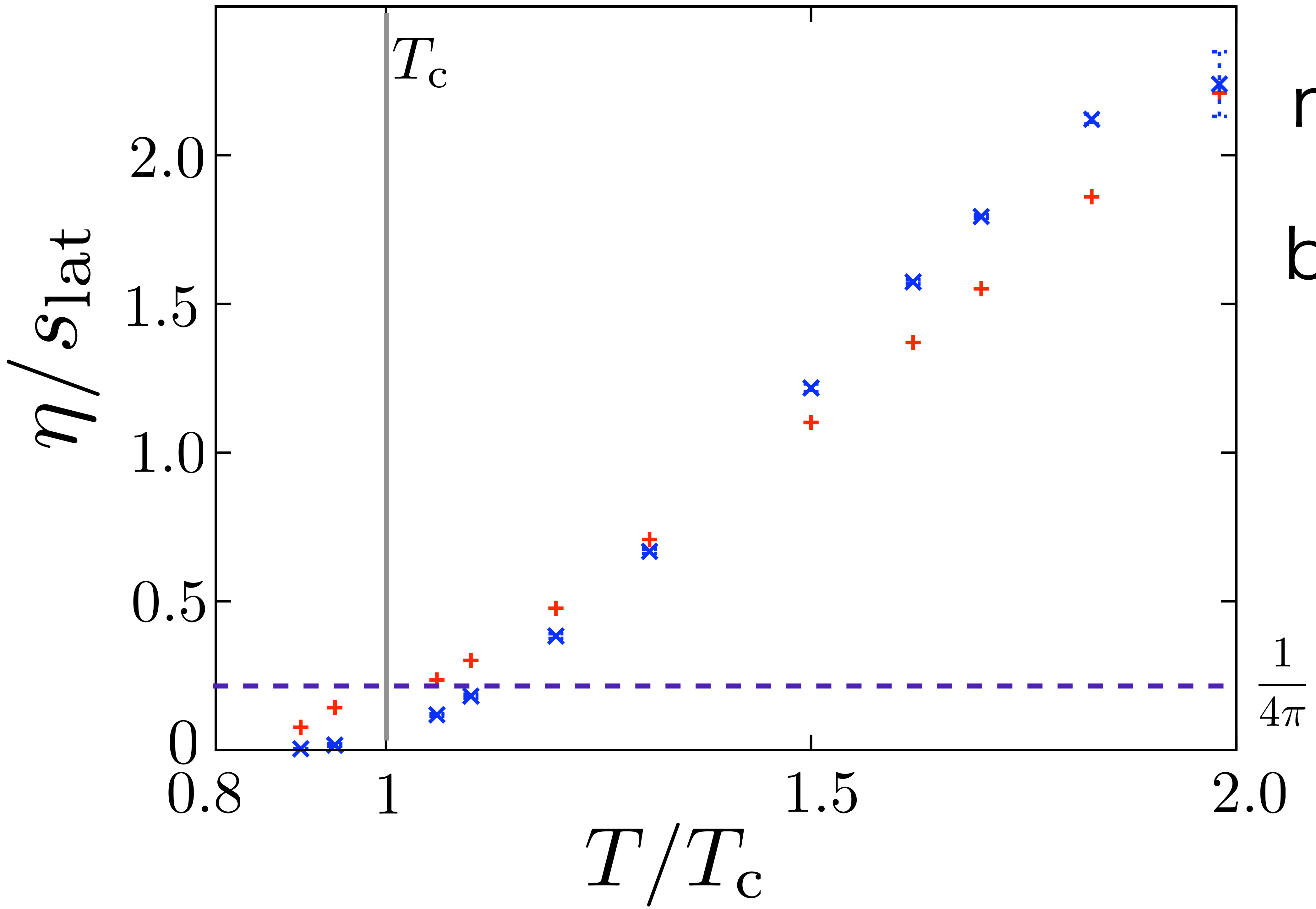
Lattice data from Cheng, et.al. PRD77,014511(2008)

Viscosity/Entropy



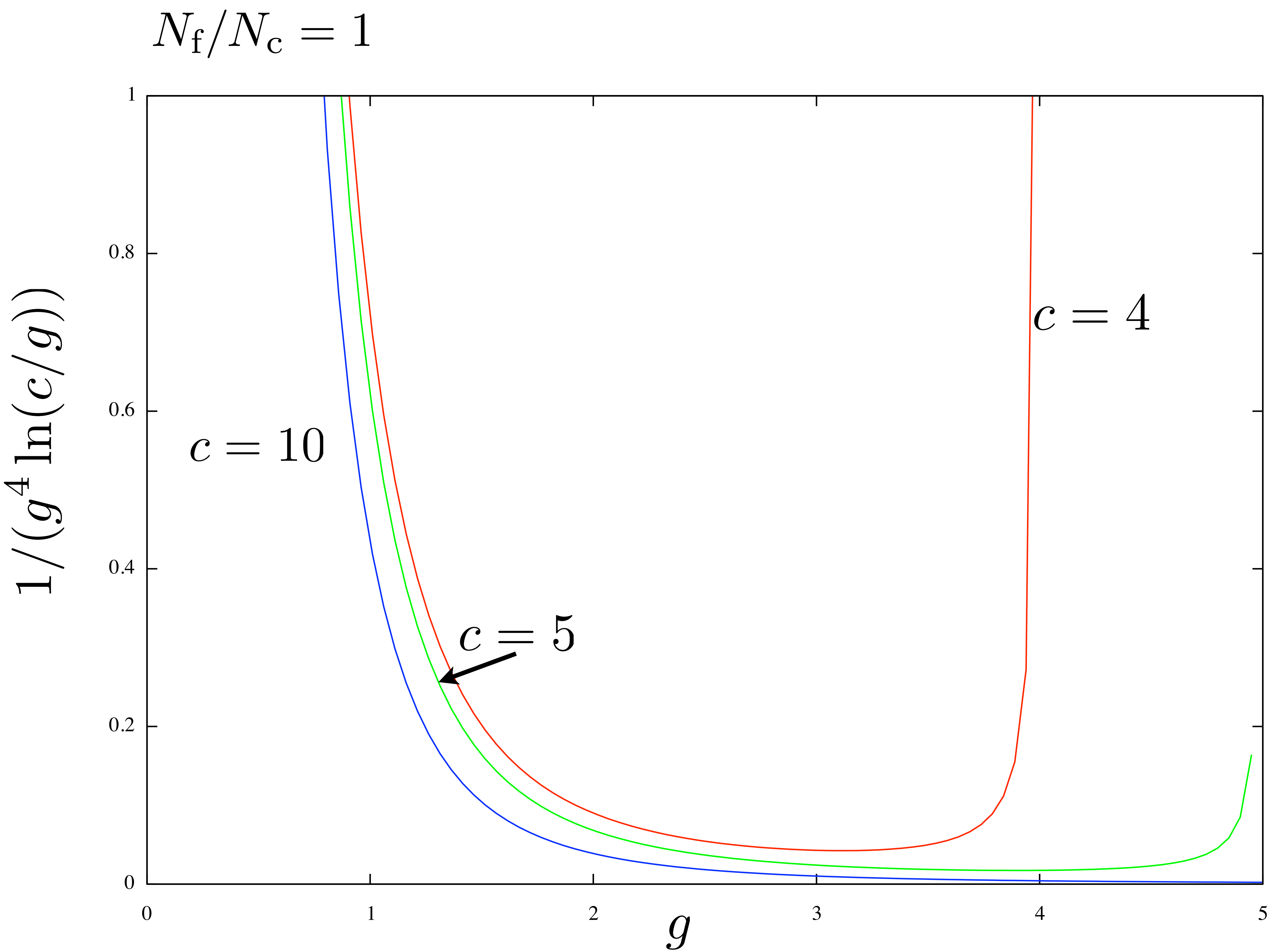
Lattice data from Cheng, et.al. PRD77,014511(2008)

Viscosity/Entropy



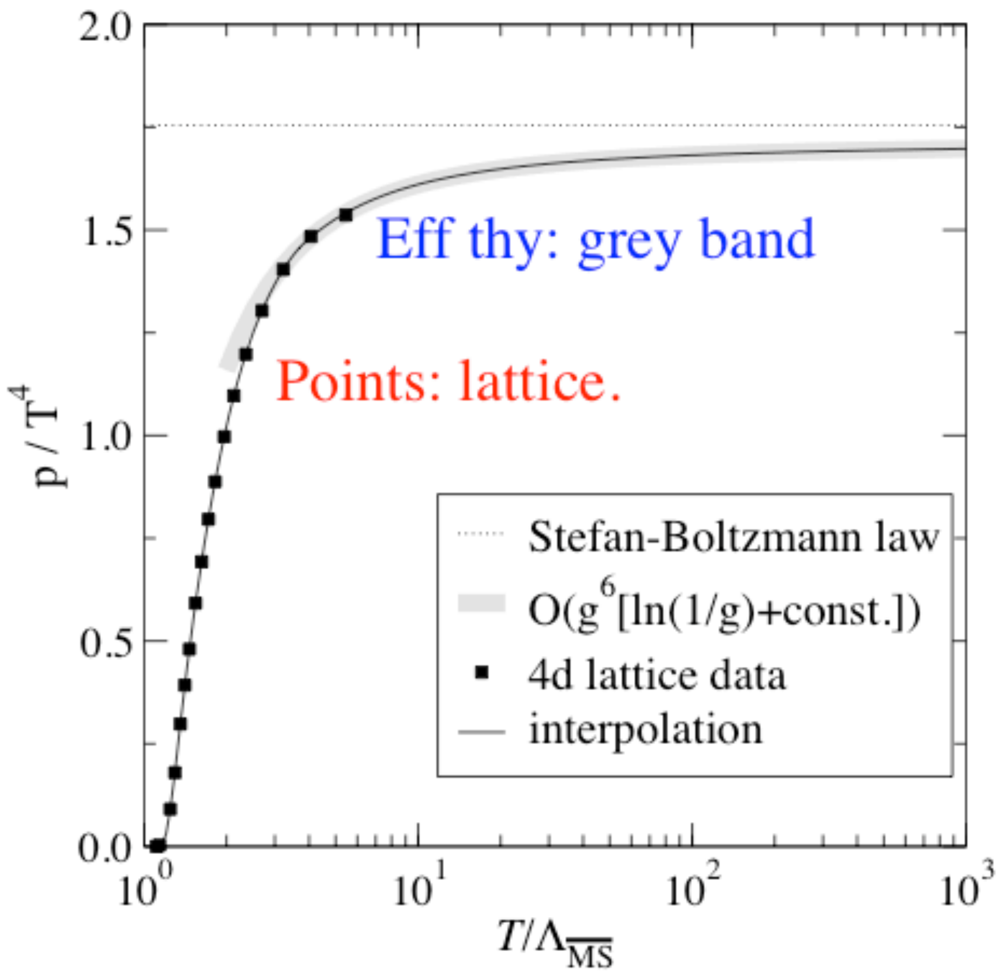
red:AMY

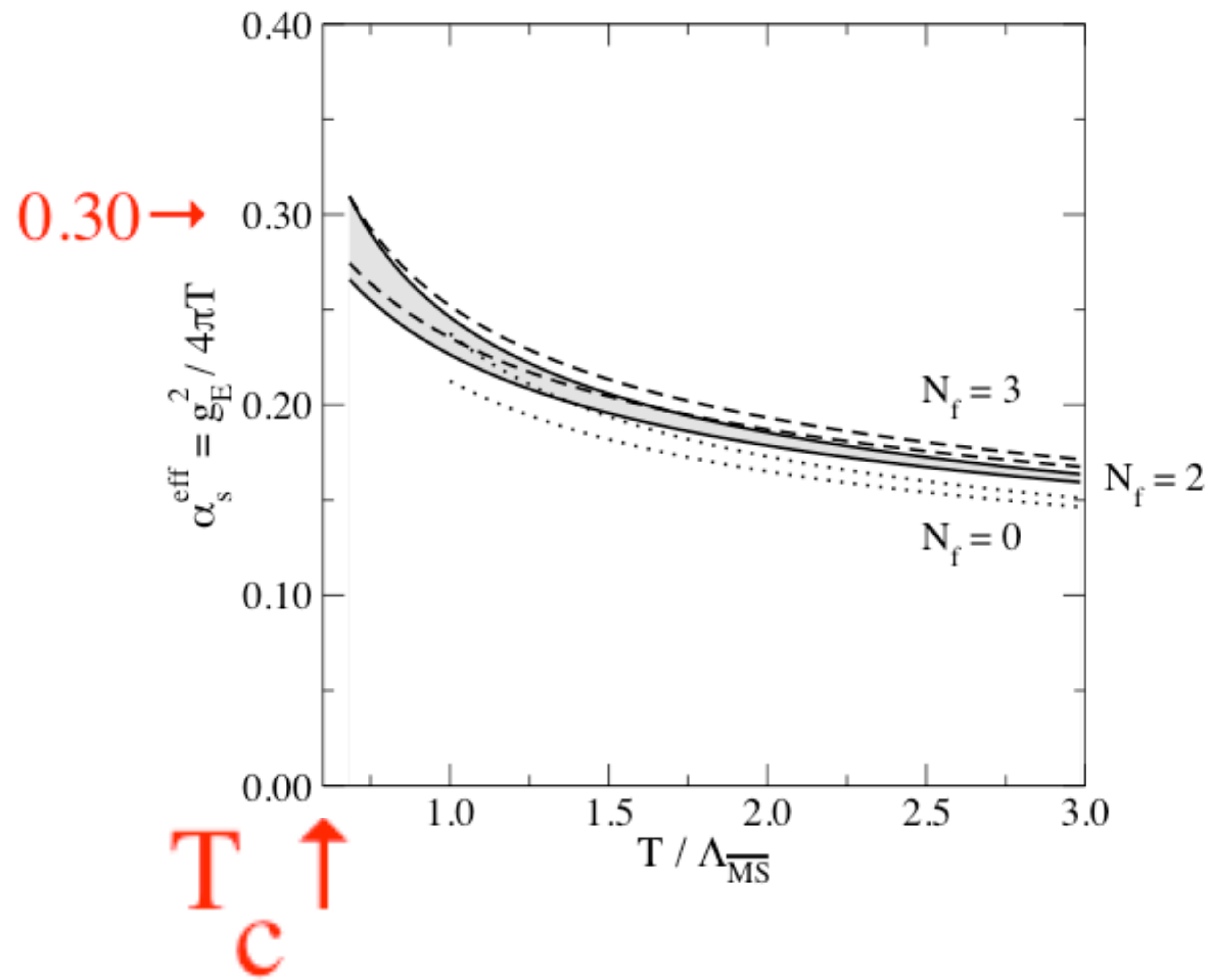
blue:ours



$N_f/N_c = 1$
$1/(g^4 \ln(c/g))$
g
$c = 10$
$c = 5$
$c = 4$
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

Weak coupling v.s. Strong coupling



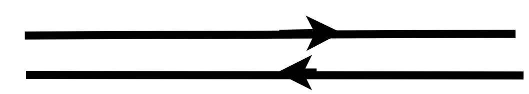


Laine &
Schröder '05

Propagator in the Semi-QGP

Quarks and gluons propagate in the background field $A_0 = Q/g$.

We use the ‘t Hooft basis (gluon =double line, quarks=single line) at finite Nc.

	Quarks	Gluons
★ Covariant derivative	$iD_0\psi^a = (k_0 + Q^a)\psi^a$	$iD_0 A_\mu^{ab} = (k_0 + Q^a - Q^b)A_\mu^{ab}$
★ Double line notation	Q^a 	$Q^a - Q^b$ 
★ Propagator	$\dfrac{1}{(k_0 + Q^a)^2 + k^2}$	$\dfrac{1}{(k_0 + Q^a - Q^b)^2 + k^2}$
★ Distribution function	$\dfrac{1}{\exp(\omega - iQ^a) + 1}$	$\dfrac{1}{\exp(\omega - iQ^a + iQ^b) - 1}$

Analytical continuation: $ik_0 + iQ^a \to \omega \pm i\epsilon$

Furuuchi(‘06)

Q corresponds to imaginary chemical potential.

Picture of Semi-QGP

★ Decompose Wilson loop to rotation of gauge invariant eigenvalue, Q.

$$\mathrm{L} = \mathrm{P} \mathrm{e}^{ig\int d\tau A_0} = \Omega^\dagger \mathrm{e}^{iQ/T} \Omega$$

★ Integrate over A_μ at fixed Q

$$Z = \int \mathcal{D}A_\mu \exp(-S[A_\mu]) = \int \mathcal{D}Q \exp(-N_\mathrm{c}^2 S_\mathrm{eff}[Q])$$

★ Integrate over Q. Valid as saddle-point at infinite-N_c

$$\frac{\delta}{\delta Q(x)} S_\mathrm{eff} = 0$$

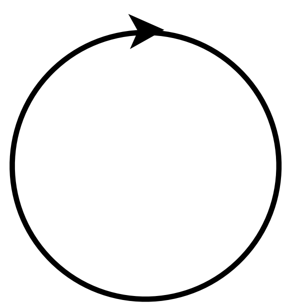
⇨ eigenvalue-distribution function $\rho(\theta)$ with $\theta(a) = Q^a/T$

$$\frac{1}{N_\mathrm{c}} \mathrm{tr}\mathrm{L}^n = \frac{1}{N_\mathrm{c}} \sum_a e^{in\theta^a} = \int da e^{i\theta(a)} = \int d\theta \rho(\theta) e^{in\theta}$$

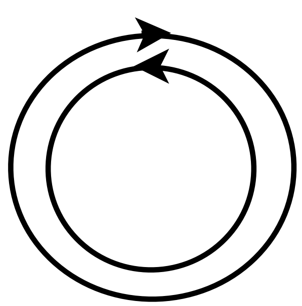
- Expand the distribution function

$$\frac{1}{\mathrm{e}^{(E-iQ^a)/T}+1}=\sum_{n=1}^{\infty}(-)^{n+1}\mathrm{e}^{-n(E-iQ^a)/T}$$

- Example: trace of the propagator



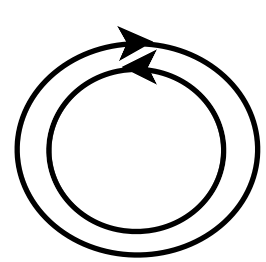
$$\sum_a\frac{1}{\mathrm{e}^{(E-iQ^a)/T}+1}=\sum_{n=1}^{\infty}(-)^{n+1}\mathrm{e}^{-nE/T}\mathrm{trL}^n$$

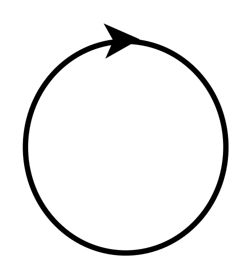


$$\sum_{a,b}\frac{1}{\mathrm{e}^{(E-i(Q^a-Q^b))/T}-1}=\sum_{n=1}^{\infty}\mathrm{e}^{-nE/T}|\mathrm{trL}^n|^2$$

Pressure(leading order)

$$P=\frac{T^4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^4}\left(2\left(|\mathrm{trL}^n|^2-1\right)+4N_{\mathrm{f}}(-1)^{n+1}\mathrm{RetrL}^n\right)$$





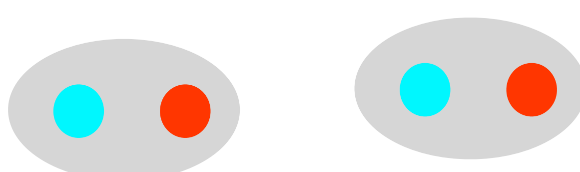
Introduction

Confinement-deconfinement Phase Transition ⟺ Ionization of color charge

Ionization parameter: Polyakov loop

$$\mathrm{trL} = \mathrm{tr}\ \mathrm{P}e^{ig\int d\tau A_0}$$

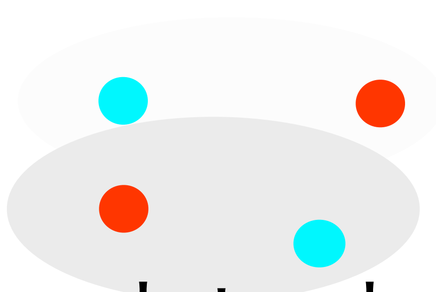
★ Confinement



No ionization

$$\left\langle \frac{1}{N_\mathrm{c}} \mathrm{tr} L \right\rangle \simeq 0$$

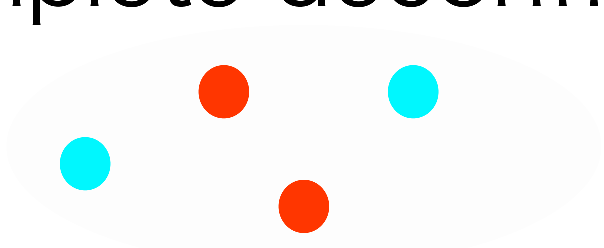
★ Partial deconfinement



Partial ionization

$$\left\langle \frac{1}{N_\mathrm{c}} \mathrm{tr} L \right\rangle < 1$$

★ Complete deconfinement



Complete ionization

$$\left\langle \frac{1}{N_\mathrm{c}} \mathrm{tr} L \right\rangle \simeq 1$$

Assumption

- ★ A_0 is decomposed to background and quantum field, $A_0 = Q/g + A_0^{\mathrm{qu}}$
- ★ Coupling is small $g \ll 1$
- ★ Background gauge field is hard $Q \sim T$
- ★ Slowly changing $\partial Q/T \sim gT$

can use derivative expansion.

Analytical continuation

Q corresponds imaginary chemical potential,

$$i\omega_n + iQ^a \rightarrow p_0 \pm i\epsilon$$

ω_n :Matsubara frequency

Viscosities

- Stress tensor

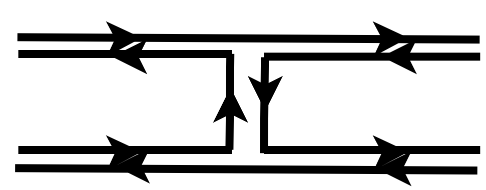
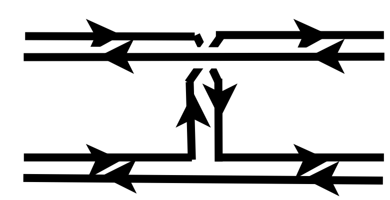
$$\langle T_{ij} \rangle = \delta_{ij} \langle \mathcal{P} \rangle - \eta\sqrt{6} X_{ij} - \xi \delta_{ij} \nabla^l u_l$$

$$X_{ij} = \frac{1}{\sqrt{6}} [\nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla^l u_l]$$

In kinetic theory

$$\langle T_{\mu\nu}(x) \rangle = \sum_{\text{spin,flavor,color}} \int \frac{d^3p}{(2\pi)^3} \frac{p_\mu p_\nu}{2\epsilon} f^a(p,x)$$

- Scattering amplitude

Pure glue $\mathcal{M} =$  $+$  $+ \cdots$

Shear Viscosity

Arnold, Moore, Yaffe (01)

Linearized Boltzmann Equation

Assume that the system is near (global) equilibrium.

Expand the distribution function:

$$f^a = f_0^a + \frac{\partial f_0^a}{\partial \epsilon} X_{ij} I_{ij} \chi^a \qquad I_{ij} = \sqrt{\frac{3}{2}}(\hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij})$$

$$f_0^a = \frac{1}{\mathrm{e}^{(u_\mu(x)p^\mu(x) - iQ^a(x))/T(x)} \pm 1}$$ in (local) equilibrium

Linear equation is obtained as

$$S = C\chi$$

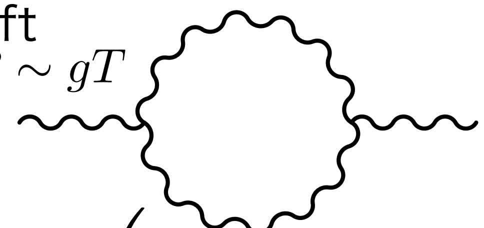
S and $C\chi$ correspond to Df and the collision term in Boltzmann equation, respectively.

The solution is formally obtained: $\chi = C^{-1}S$

$$\Rightarrow \quad \eta = \frac{1}{15} S^t C^{-1} S$$

HTL's in the semi-QGP

Hard Thermal Loop approximation

$\Pi^{\mu\nu}(P) =$  $+$ Tadpole, ghost diagrams

$$= (m_D^2)^{ab}\left(\delta^{\mu 0}\delta^{\nu 0} - \int \frac{d\Omega}{4\pi}\frac{(P)^0 \hat{K}^\mu \hat{K}^\nu}{P\cdot\hat{K}}\right) - i f^{ab}{}_c \langle (J^c)^0 \rangle \int \frac{d\Omega}{4\pi}\frac{\hat{K}^\mu \hat{K}^\nu}{P\cdot\hat{K}}$$

Ordinary HTL approx. term

The thermal mass changes.

$\boxed{\text{Debye mass}}$ $\quad [m_D^2(Q)]^{ab} = m_D^2 \times h^{ab}(Q)$ $\quad$ where $m_D^2 = \frac{1}{6} N_c g^2 T^2$

- ★ Complete QGP $\quad h^{ab} = \delta^{ab}$
- ★ Semi-QGP phase $\quad h^{ab} < 1$
- ★ Confined phase $\quad h^{ab} \sim 0$ $\quad$ gluons don't propagate.

Viscosities

Arnold, Moore, Yaffe (01)

Linearized Boltzmann Equation

Distribution function $f^a = f_0^a + \frac{\partial f_0^a}{\partial \epsilon} f_1 \qquad f_1 \ll 1$

$$D f^a = \frac{\partial f_0^a}{\partial \epsilon} |\boldsymbol{p}| I_{ij} X_{ij} \qquad I_{ij} = \sqrt{\frac{3}{2}} (\hat{p}_i \hat{p}_j - \frac{1}{3}\delta_{ij})$$

$$C^a[f] = \frac{1}{2} \sum_{\text{color,spin,flavor}} \int d\Pi (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2$$
$$\times f_0^a f_0^b (1 \pm f_0^c)(1 \pm f_0^d)(f_1^a + f_1^b - f_1^c - f_1^d)$$

Distribution function

$$X_{ij} = \frac{1}{\sqrt{6}} [\nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla^l u_l]$$

$$f_0^a = \frac{1}{\mathrm{e}^{(u_\mu(x) p^\mu(x) - iQ^a(x))/T(x)} \pm 1}$$ in (local) equilibrium

$$S = C\chi$$