Dynamic aspects of the QCD critical point

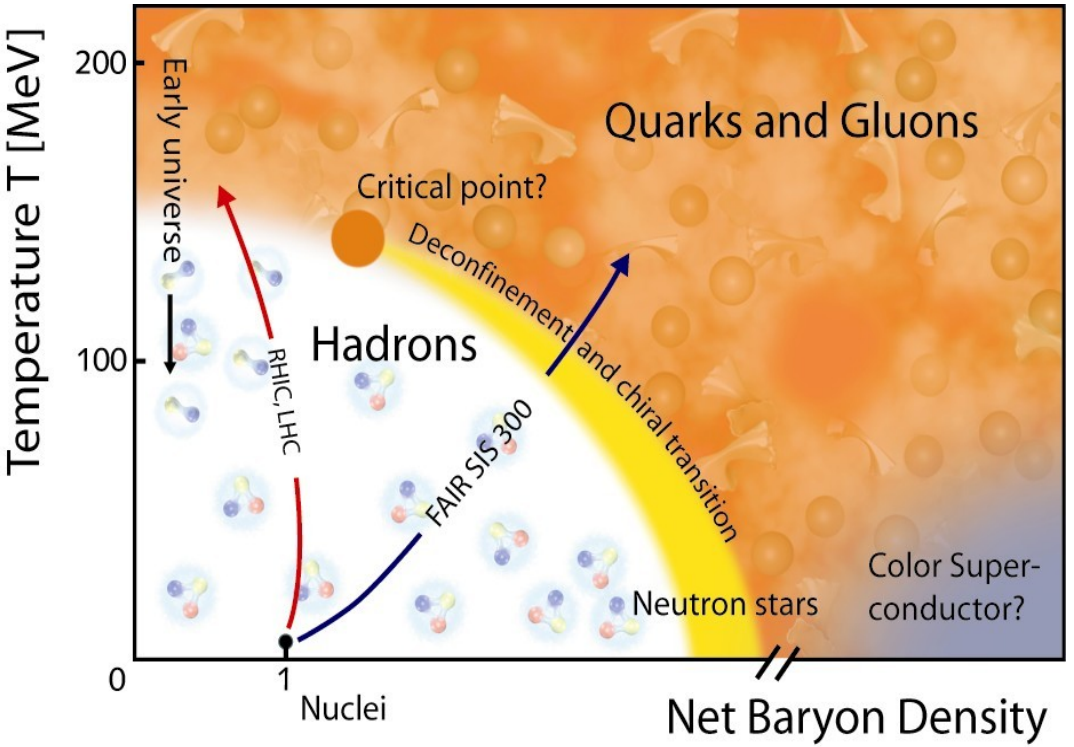
Hiro Fujii

HF, PRD67, 094018
HF-M.Ohtani, PRD70, 014016
and work in progress

Introduction

- **LQCD + Models suggested the QCD-CP in T-μ plane**
  - **cross-over along T axis**
  - **1st order along mu axis**

Stephanov-Rajagopal-Shuryak
PRL 81, 4816 (98); PRD60,114028(98)



Introduction

- LQCD + Models suggested the QCD-CP in T-μ
  - cross-over along T axis
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- Still difficult to conclude with LQCD+Models today
  - about the location (even the existence?)

For review, Stephanov
Fodor-Katz, Ejiri et al, de Forcrand-Philipsen

- For exp'tal confirmation, we need good signatures
- Important to clarify the character of the QCD-CP

How to use chiral models

Stephanov et al., Hatta-Ikeda, HF-Ohtani, Sasaki et al.

- Generic features of the CP can be demonstrated in simple models
  - singular behavior of fluctuations, ..., etc.

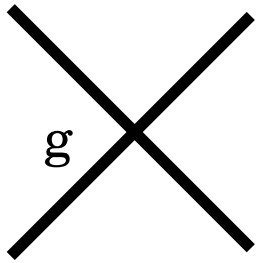
- This talk: spectral properties near the QCD-CP
  - within a simple, but consistent model
  - message: 'pole' mass of σ is non-zero @QCD-CP

- Take NJL model, although t-dep Ginzburg-Landau theory may be used

HF-Ohtani, PRD70, 014016

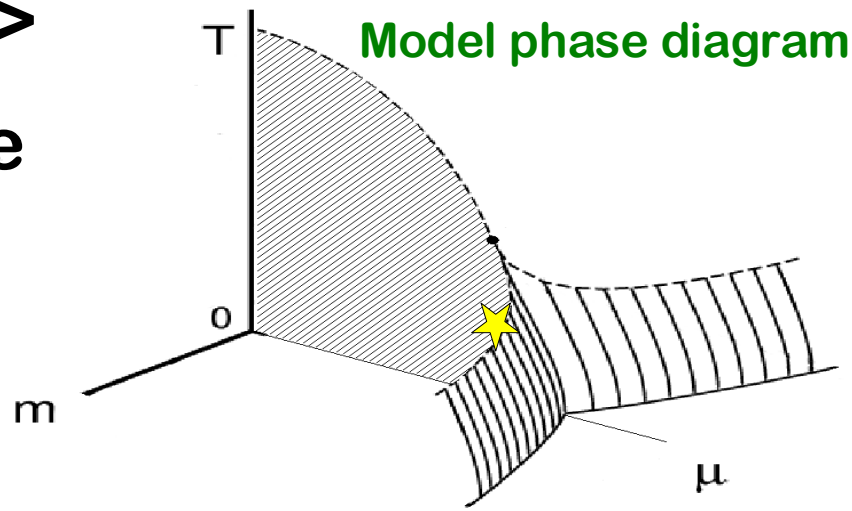
NJL model

- $$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + g[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau^a q)^2]$$



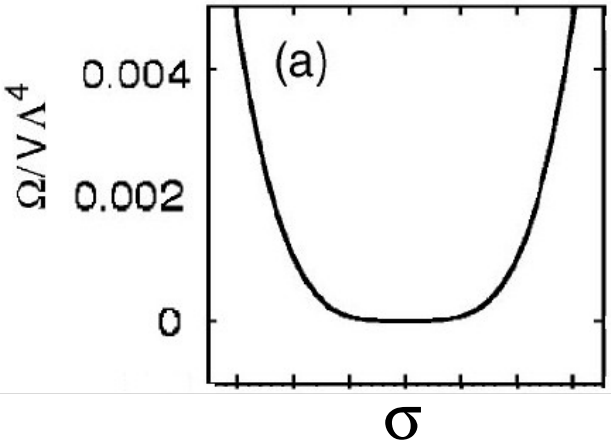
  - the simplest quark dynamics
  - chiral symmetry breaking by q^{bar}-q attraction
  - no dynamic gluons, no confinement (nucleons)
  - σ and π as fluctuations of $\langle q^{bar} q \rangle$
  - In reality, m /= 0, of course

Model phase diagram

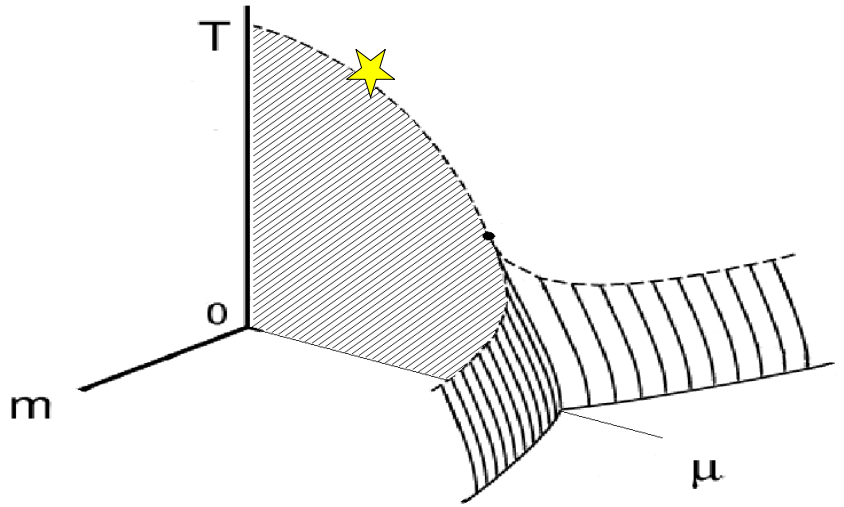


Chiral transition: warm-up

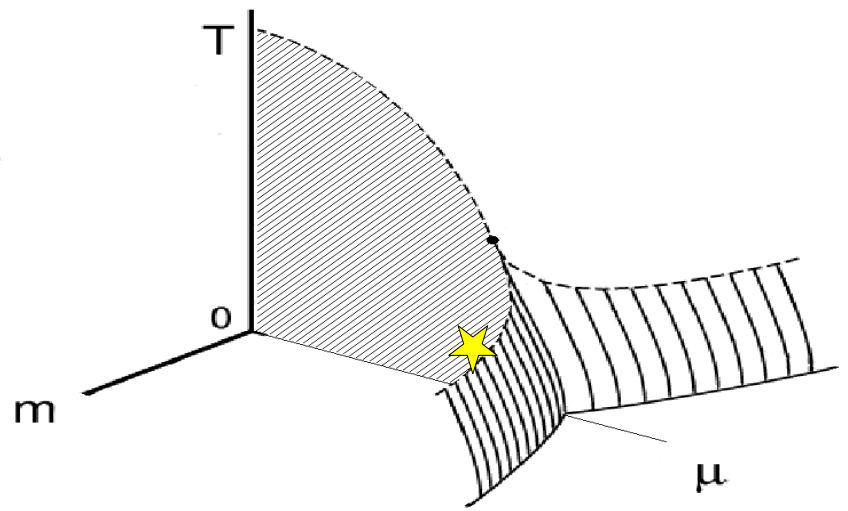
- **THE** order parameter $\langle q^{bar} q \rangle \sim \langle \sigma \rangle$
- Scalar density gap opens below CP
- chiral susceptibility: $\langle \sigma^2 \rangle - \langle \sigma \rangle^2$
  - $\chi_{mm} = -\frac{\partial^2 P}{\partial m^2} \sim \int d^3 r \frac{e^{-M_\sigma r}}{r} \sim \frac{1}{M_\sigma^2} \to \infty$
  - long-range fluctuation @CP



- $\chi_{\mu\mu}$, *C* finite (in MF)
  - Since no gap for n_B & *s*



CP with nonzero m

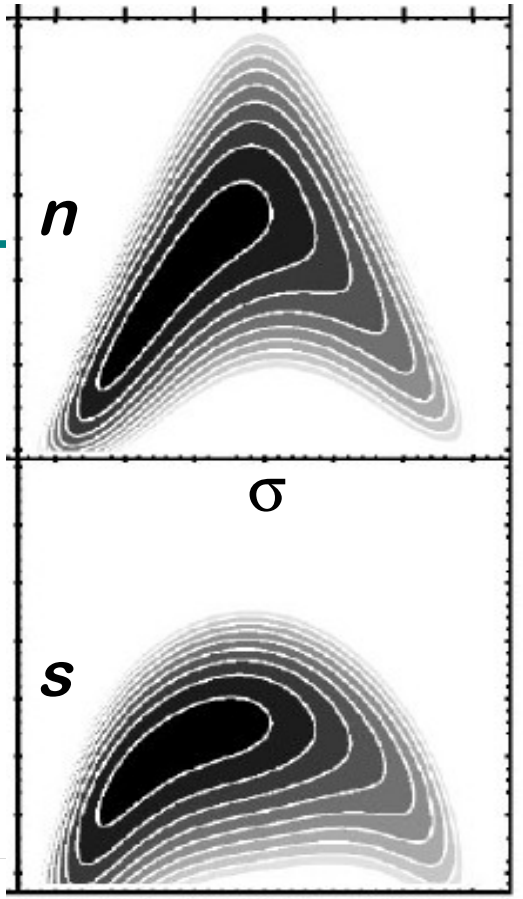


CP with nonzero m

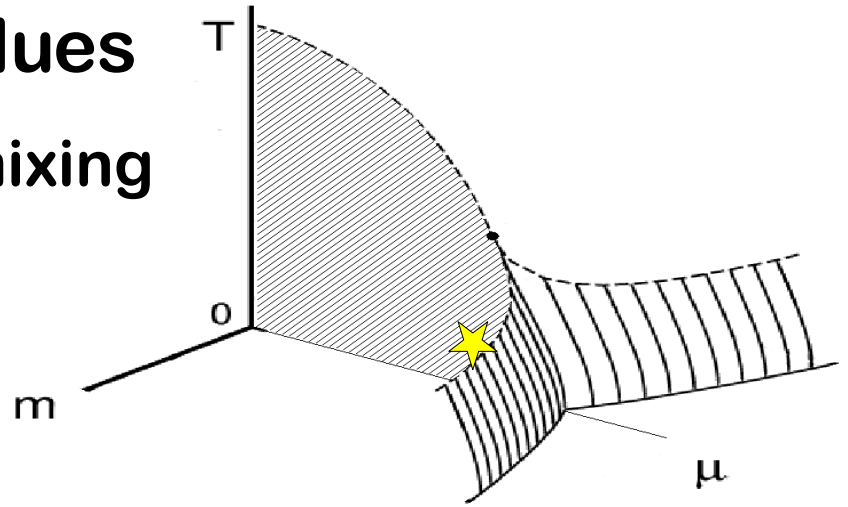
- Across co-existence surface all densities have gaps (Clapeylon-Clausius)

$$\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s}, \quad \frac{dT}{dm} = -\frac{\Delta \sigma}{\Delta s}$$

- Flat potential wrt σ, n_B, and/or *s* @CP
- order parameter = deviations of σ, n_B, and/or *s* from equilibrium values
  - Nonzero *m* generates linear mixing
- χ_{mm}, $\chi_{\mu\mu}$ and *C* all diverge

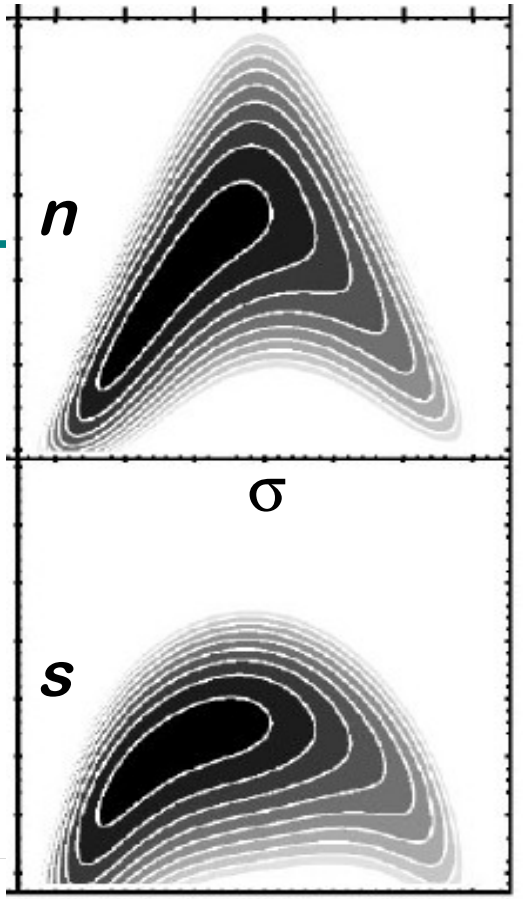


Contour plot of free energy



CP with nonzero m

- χ_{mm}, $\chi_{\mu\mu}$ and *C* all diverge
  - $\chi \sim 1/M_\sigma^2$
  - massless σ? ω? or what?



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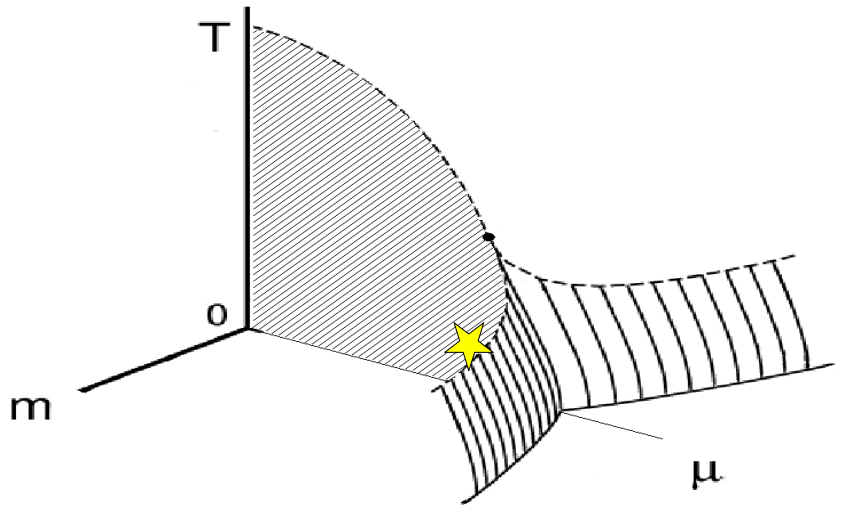

χ's & spectral change

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- q^{bar}-q attraction induces the CP
- Susceptibilities, which diverge at CP, are expressed in terms of mode spectrum

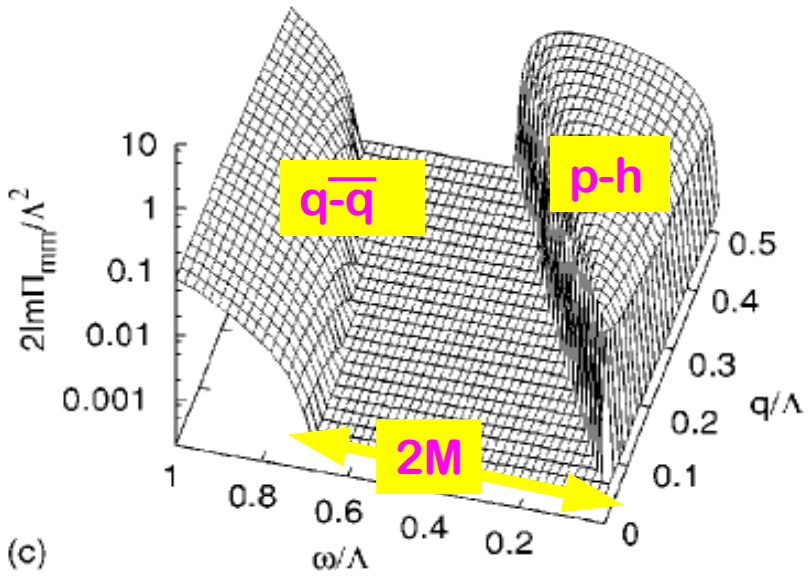
$$\chi_{ab}(q)=\int \frac{d\omega}{2\pi}\frac{\rho_{ab}(\omega,q)}{\omega}$$

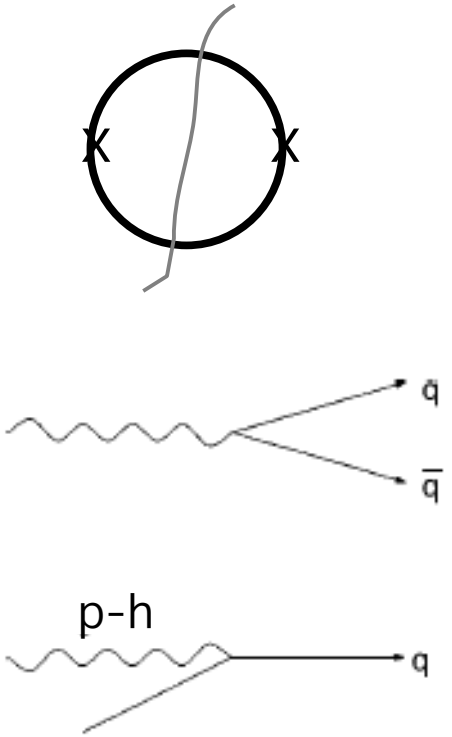
- What mode softens in $\rho_{ab}(\omega,q)$ near CP ?
- RPA calc in NJL:

$$\chi_{ab}(iq_4,\mathbf{q})=\Pi_{ab}(iq_4,\mathbf{q})+\Pi_{am}(iq_4,\mathbf{q}) \times\frac{1}{1-2g\Pi_{mm}(iq_4,\mathbf{q})}2g\Pi_{mb}(iq_4,\mathbf{q})$$

Free spectral fn. ImΠ

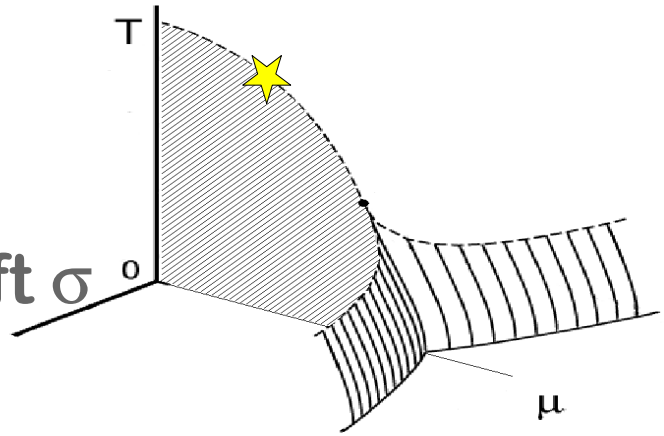
- Spectral fn of massive quark gas
  - spectrum in time- and space-like regions
  - space-like (p-h) mode is characteristic in a medium



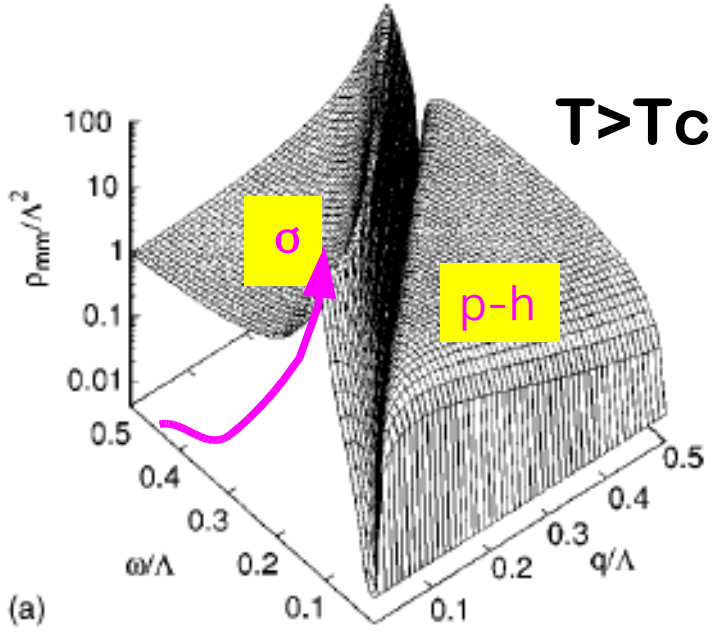


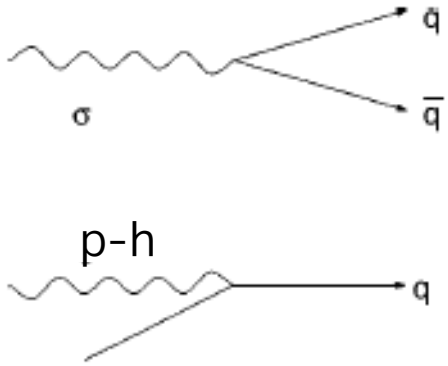
O(4)case: dropping mass

- **Common sense** near O(4) CP
  - dynamic σ-mode softens near CP
  - thermally-massive quarks give soft σ
  - no p-h mode contributes to χ_{mm}



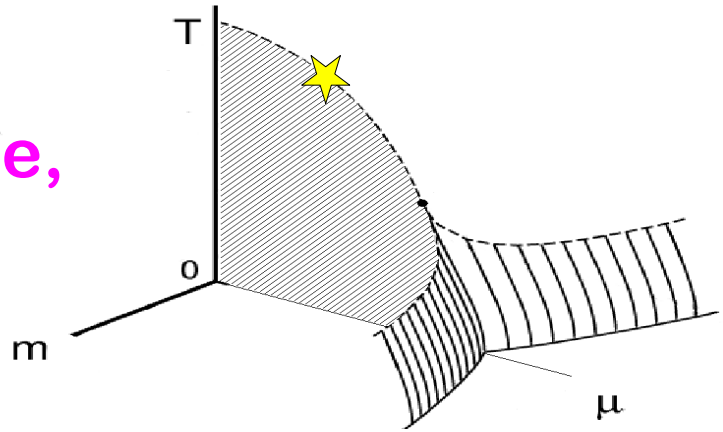
Hidaka-Kitazawa



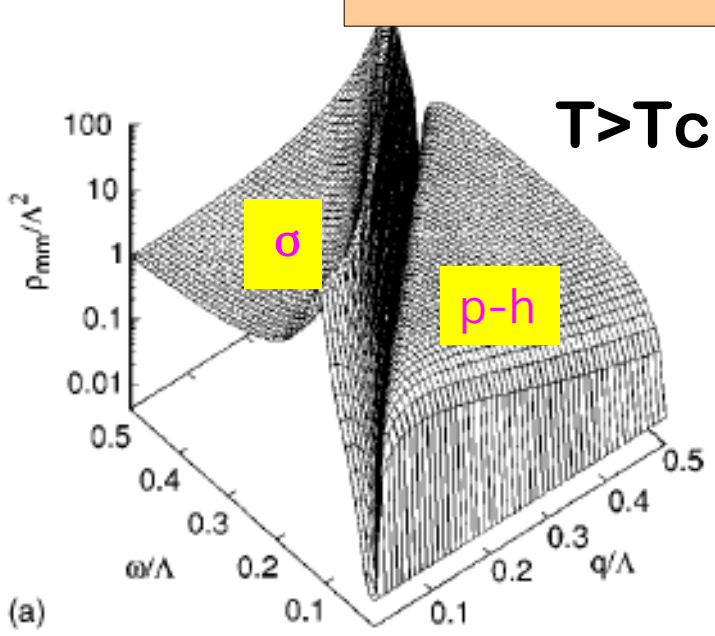


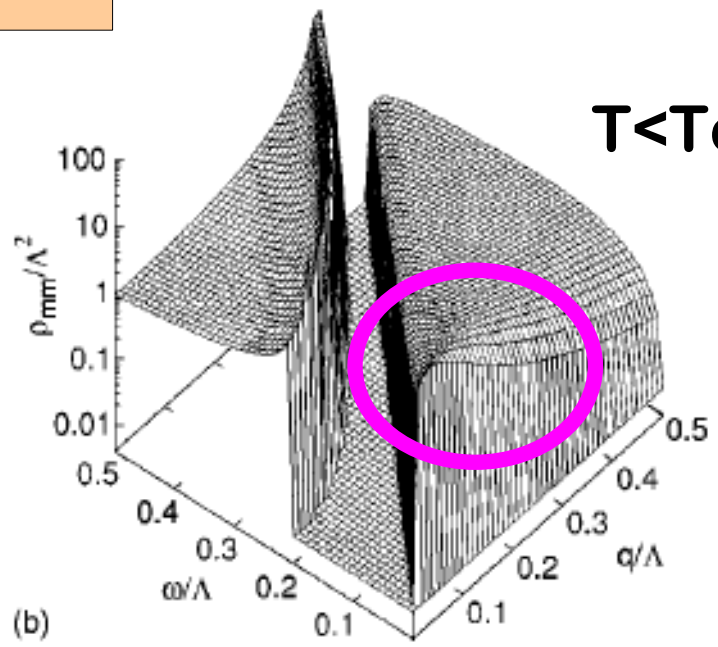
O(4)case: dropping mass & ...

- **A little surprise** near O(4) CP
  - if approached from broken phase, non-zero *M* mixes p-h in χ_{mm}



$$\chi_{ab}(q)=\int \frac{d\omega}{2\pi}\frac{\rho_{ab}(\omega,q)}{\omega}$$





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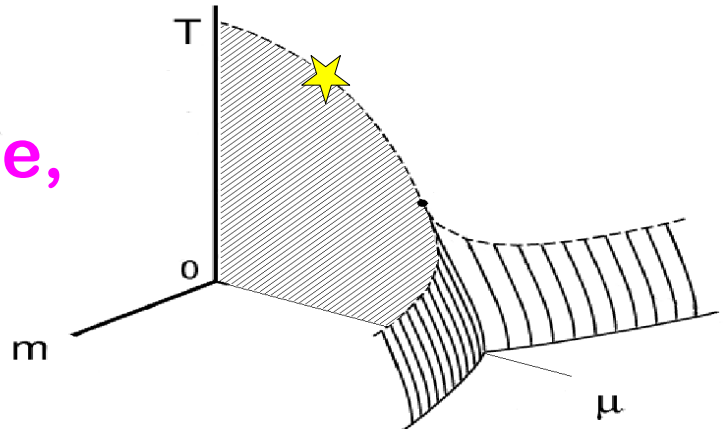

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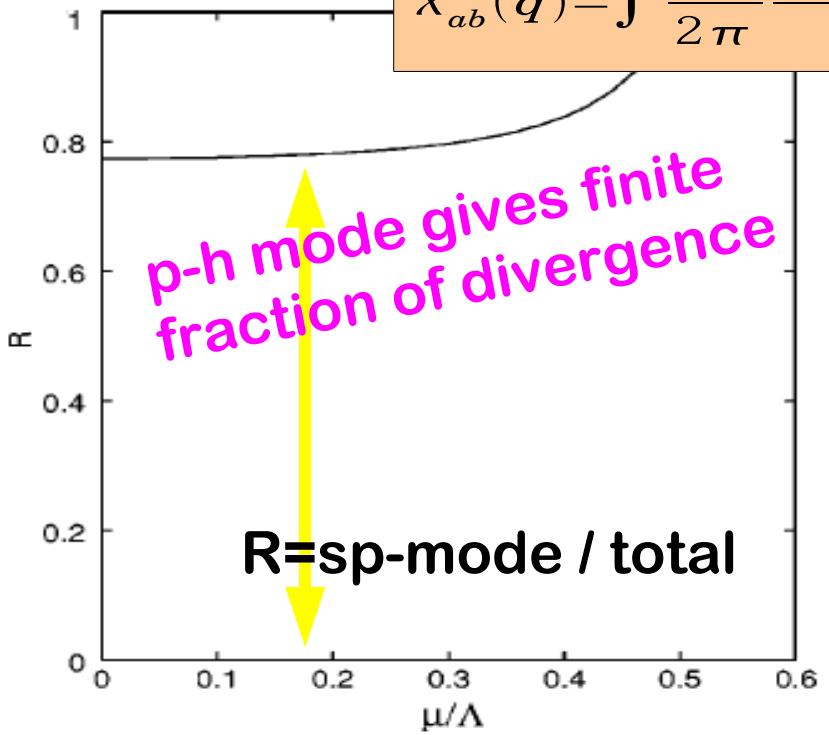

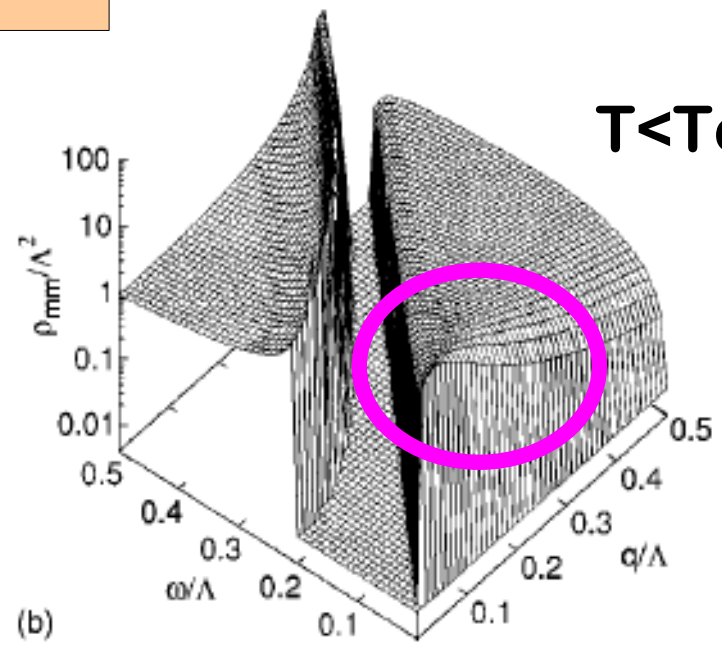

Toward TCP: what happens?

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- In addition to soft sigma,
  - p-h mode enhances and saturates the divergence !

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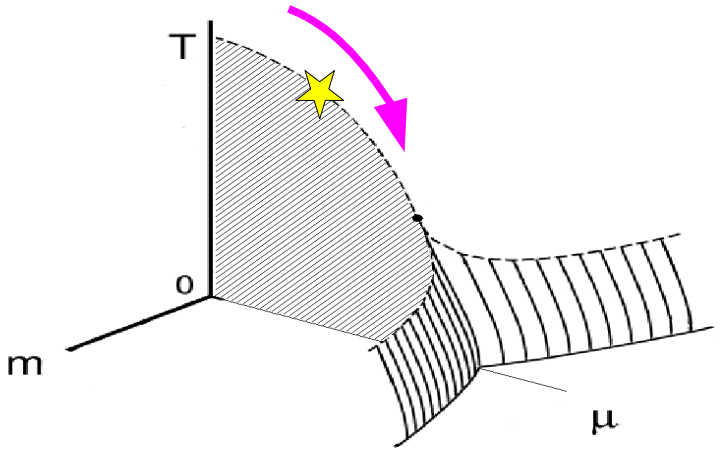
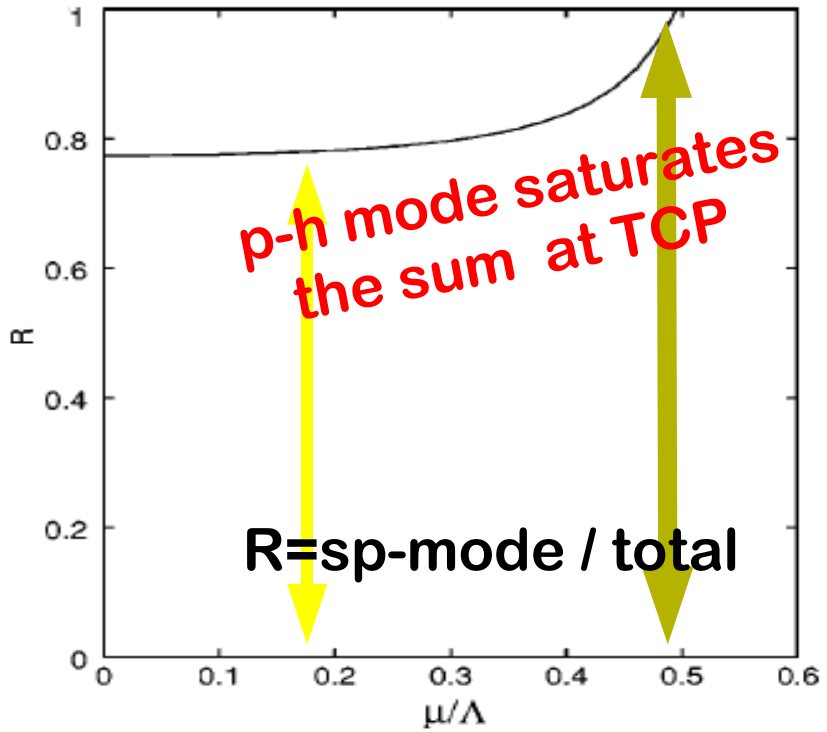
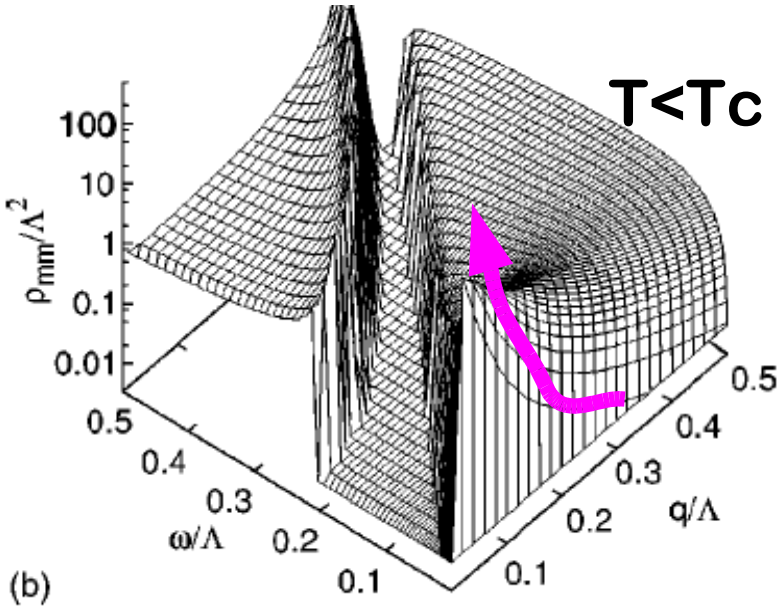
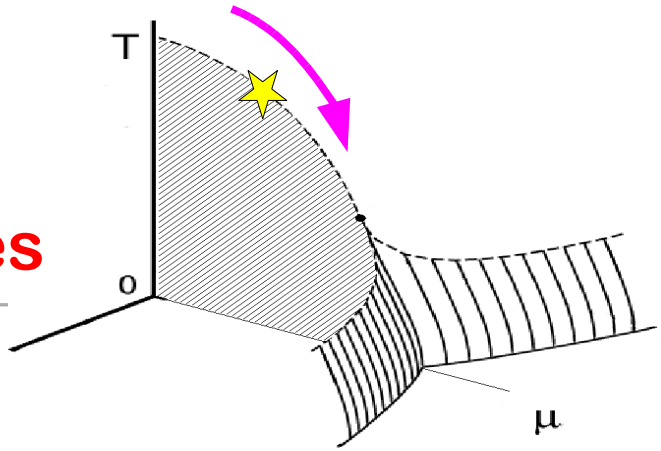


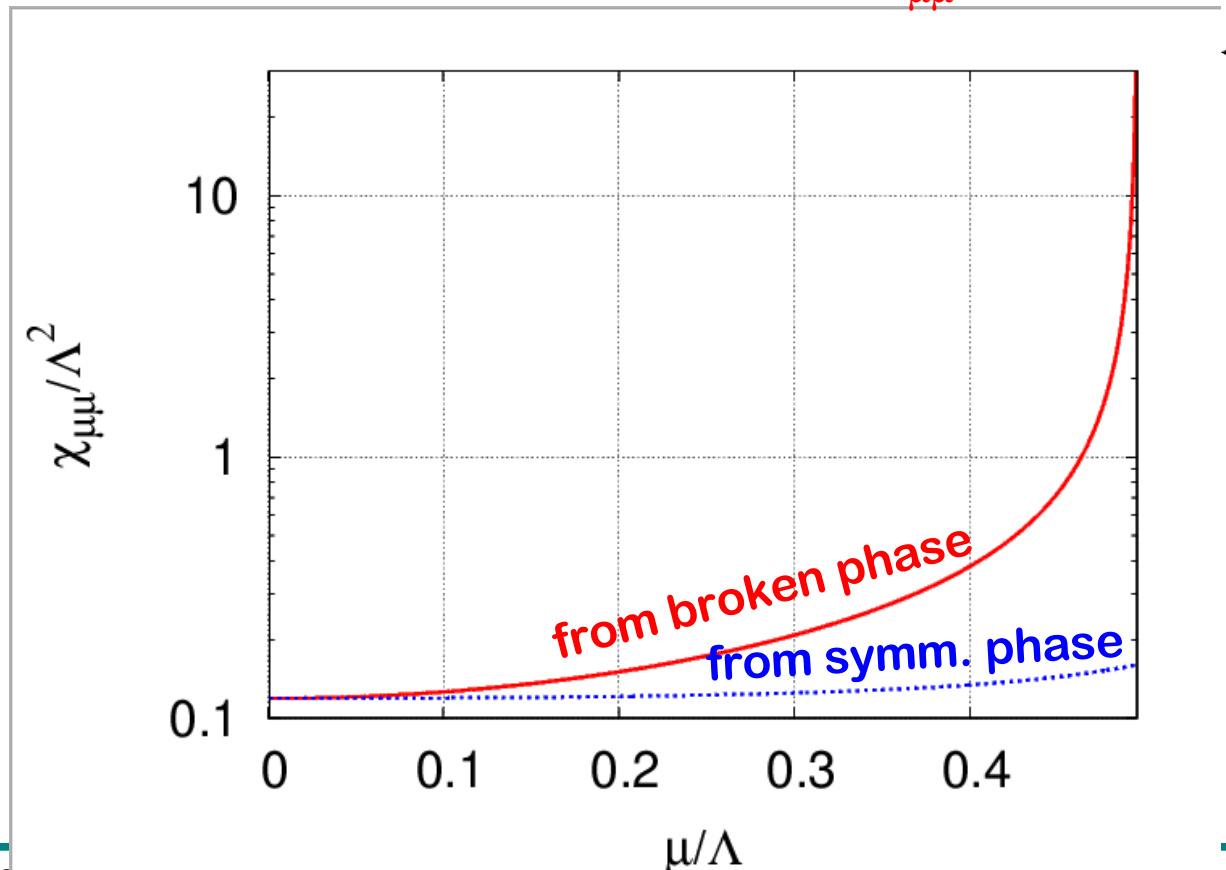




Toward TCP: what happens?

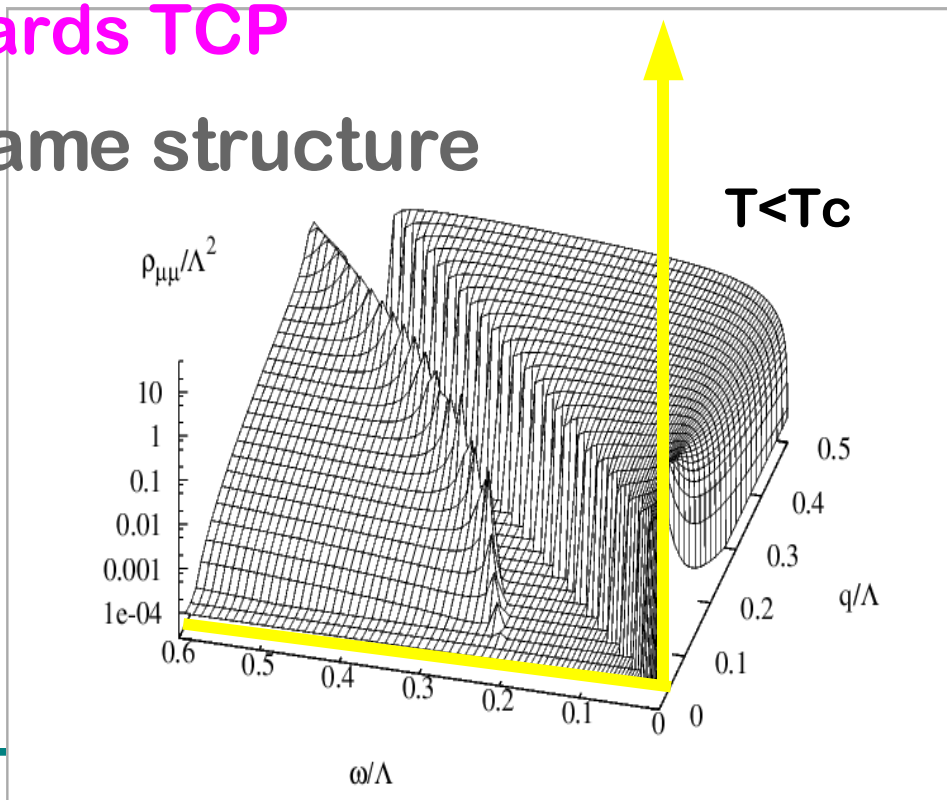
- In addition to soft sigma,
  - p-h mode enhances !
  - quark number suscept. $\chi_{\mu\mu}$ diverges



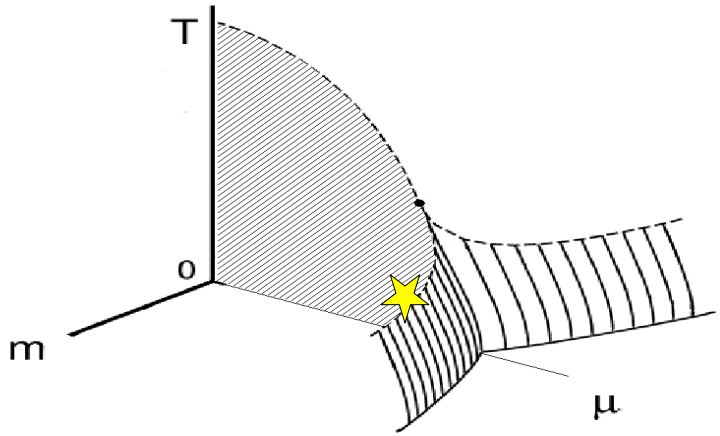


Why is p-h mode enhanced?

- $\chi_{\mu\mu}$ = fluctuation of conserved density
  - at q=0 (total charge) it cannot fluctuate
    - spectral strength only at ω=0: $\lim_{q\to 0}\rho_{\mu\mu}(\omega,q) \propto \omega\delta(\omega)$
  - the strength diverges towards TCP
  - specific heat *C* has the same structure

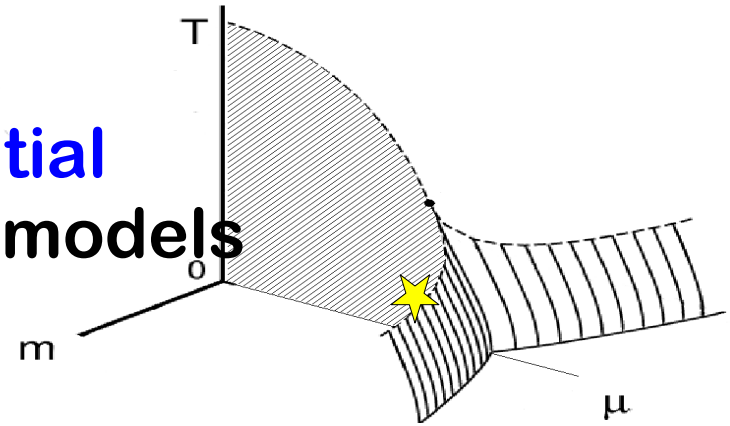


CP with nonzero m



CP with nonzero m

- mass of sigma?
  - Scavenius et al computed potential curvature & pole masses in two models



PRC64, 045202

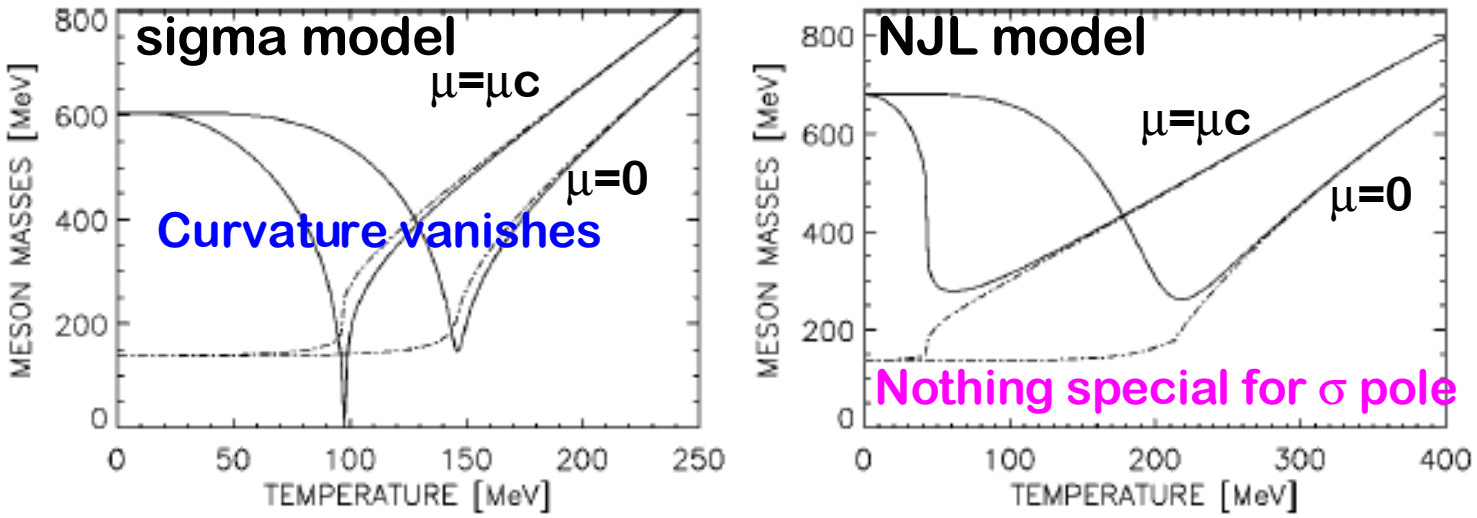
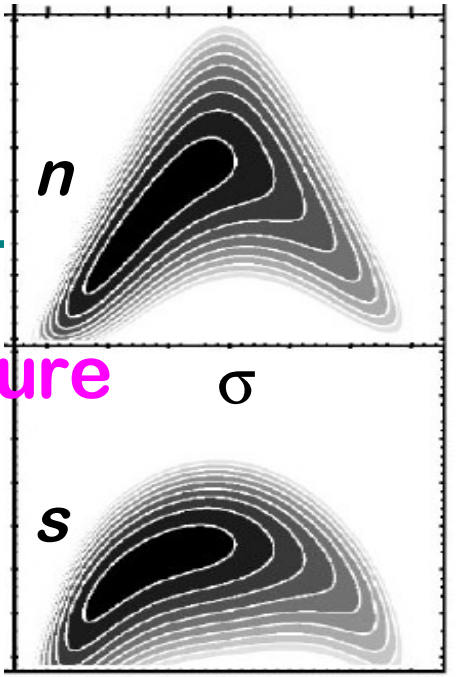


FIG. 6. The sigma mass (solid line) and pion mass (dashed line) in the sigma model (left) and NJL model (right) as functions of temperature for $\mu=0$ (right pair) and for $\mu=\mu_c$ (left pair).

CP with nonzero m

- p-h mode is responsible for flat curvature & all divergent χ's
  - no dropping mass at QCD-CP !
  - due to explicit breaking *m*



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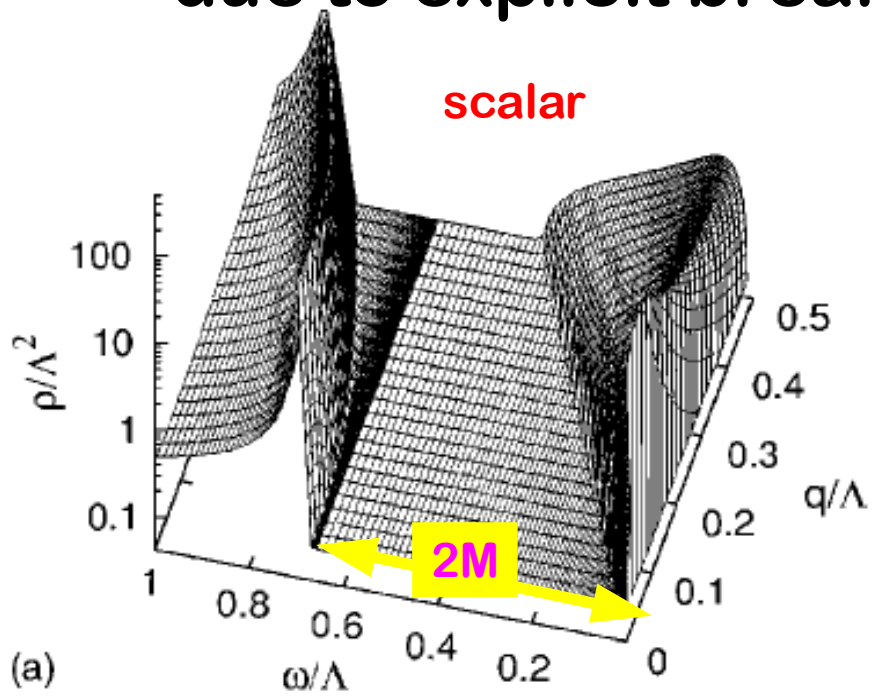
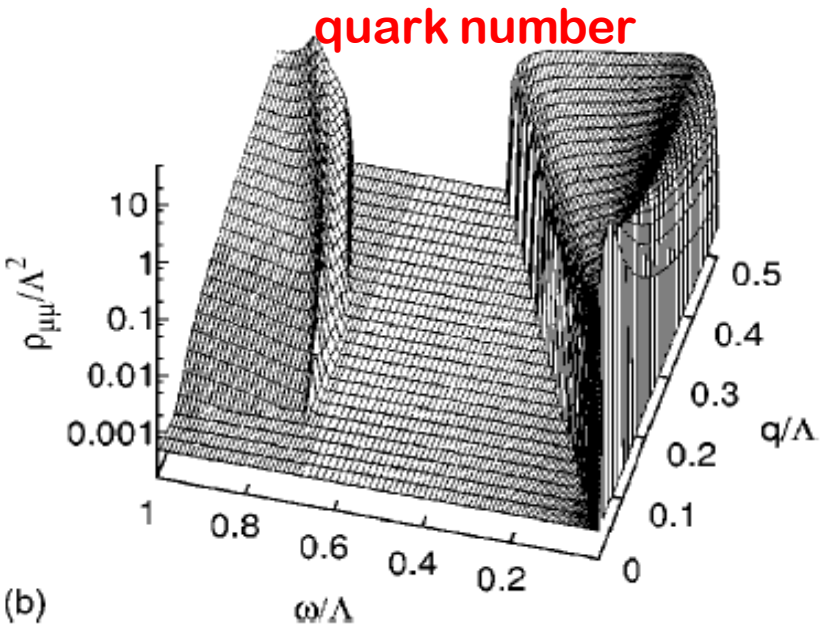




Dynamic Universality

Hohenberg-Halperin
HF-Ohtani
Son-Stephanov

- Static universality of QCD-CP
  - 3D Z(2) Ising
- **Dynamics is constrained by conservation laws**
  - **possible slow modes near QCD-CP**
    - σ (π massive due to $m_q \neq 0$)
    - $T^{0\mu}$, n ... 5 densities
  - **decoupling of σ due to finite *m***
  - **slow modes = sound (2), shear (2), heat (1) = liq.gas**
  - relativity vs dynamic universality ?

Hints to phenomenology

- critical softening in p-h mode
  - decoupling of σ and ω mesons
  - no direct access to dileptons, $\pi\pi$
  - open question on 'strong NN attraction' near CP
- within critical region of gas-liquid CP
  - shear viscosity (small exponent) $\eta \sim \xi^{(1/19)\varepsilon}$
  - thermal conductivity $\lambda \sim \xi^{(18/19)\varepsilon}$
- finite size / time effects always in HIC

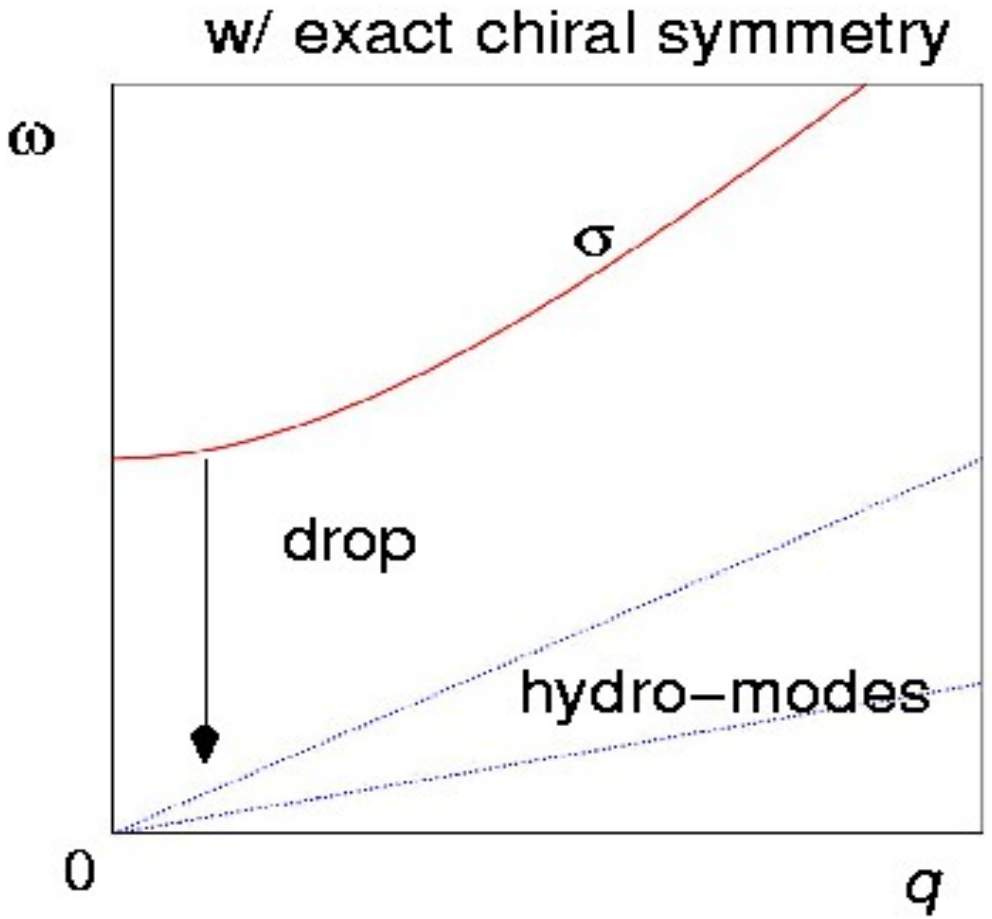
Berdnikov-Rajagopal

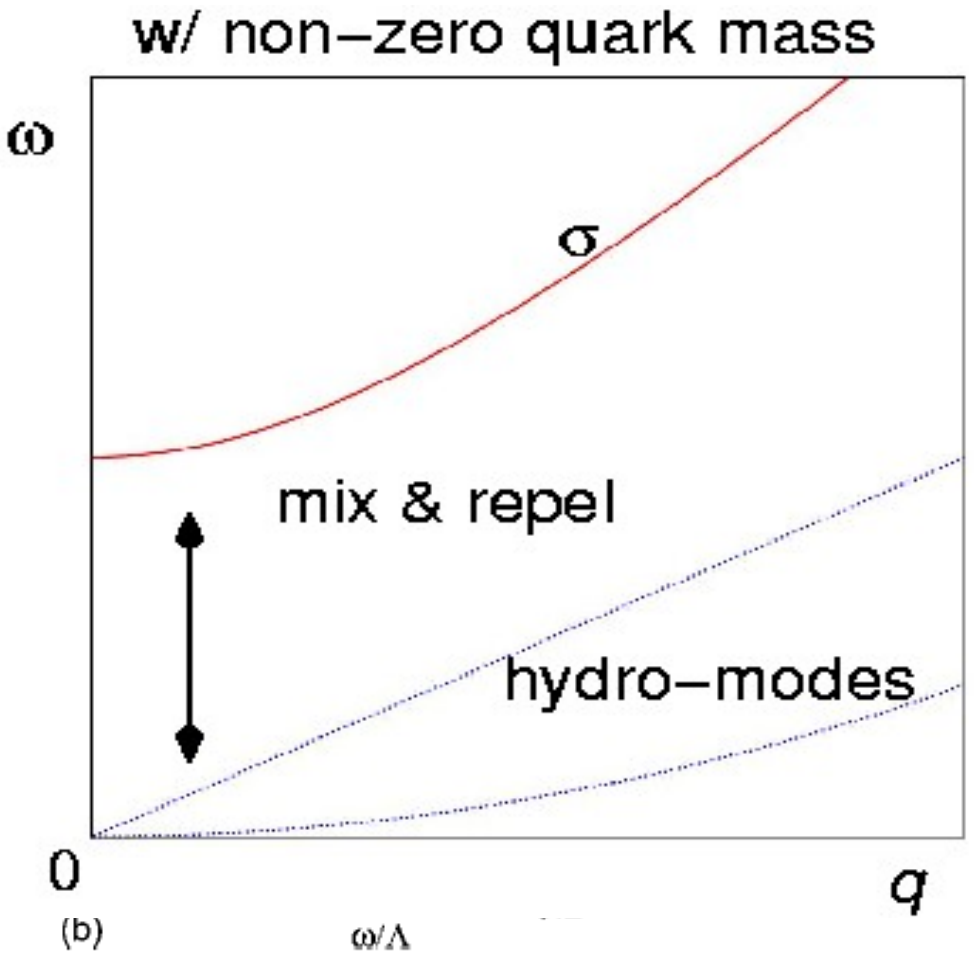
Summary

- **Simple chiral models demonstrate:**
  - **σ softens near chiral CP (as everybody knows)**
  - **p-h also contributes χ_{mm} if approached from broken phase**
  - **Towards TCP, p-h mode is enhanced to saturate the divergence**
- **Critical mode of QCD-CP has p-h type spectrum**
  - **no massless σ, no direct dilepton, $\pi\pi$ decay**
- **Dynamic Universality is the same as liq.-gas**
  - **η & λ should become large in the critical region**

Why is p-h mode enhanced?

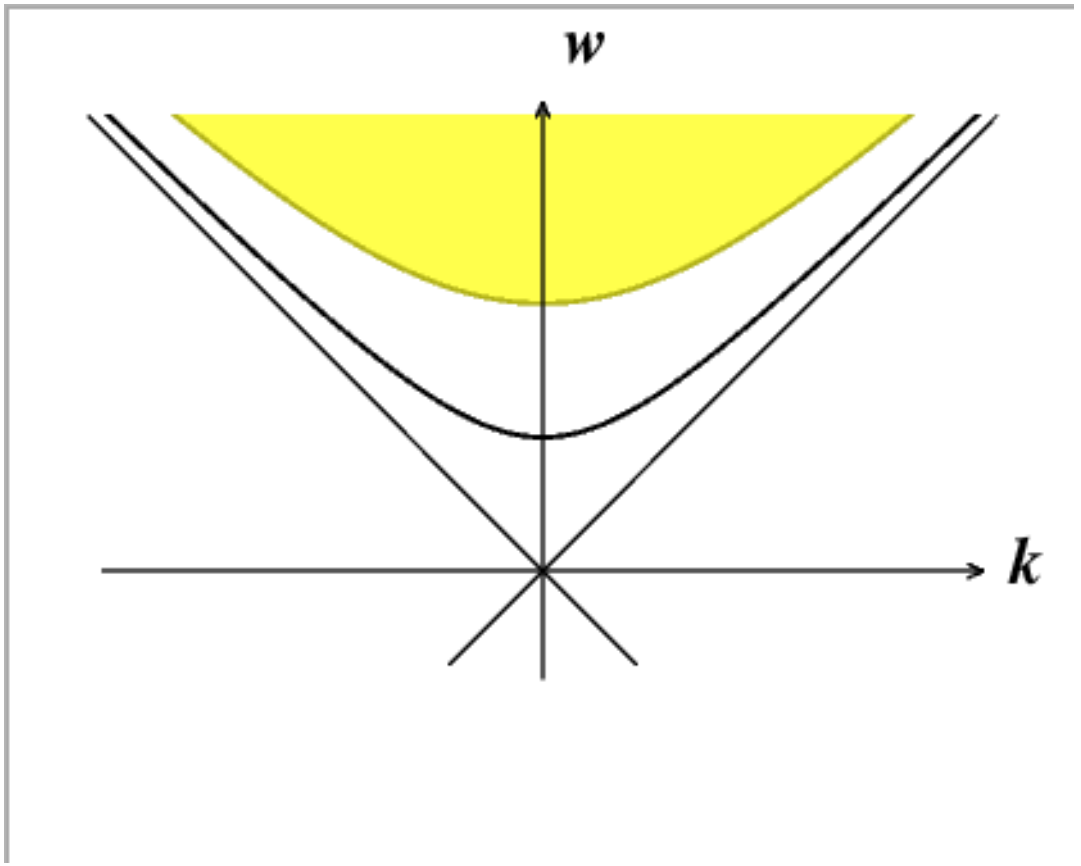
- **Schematic sketch**





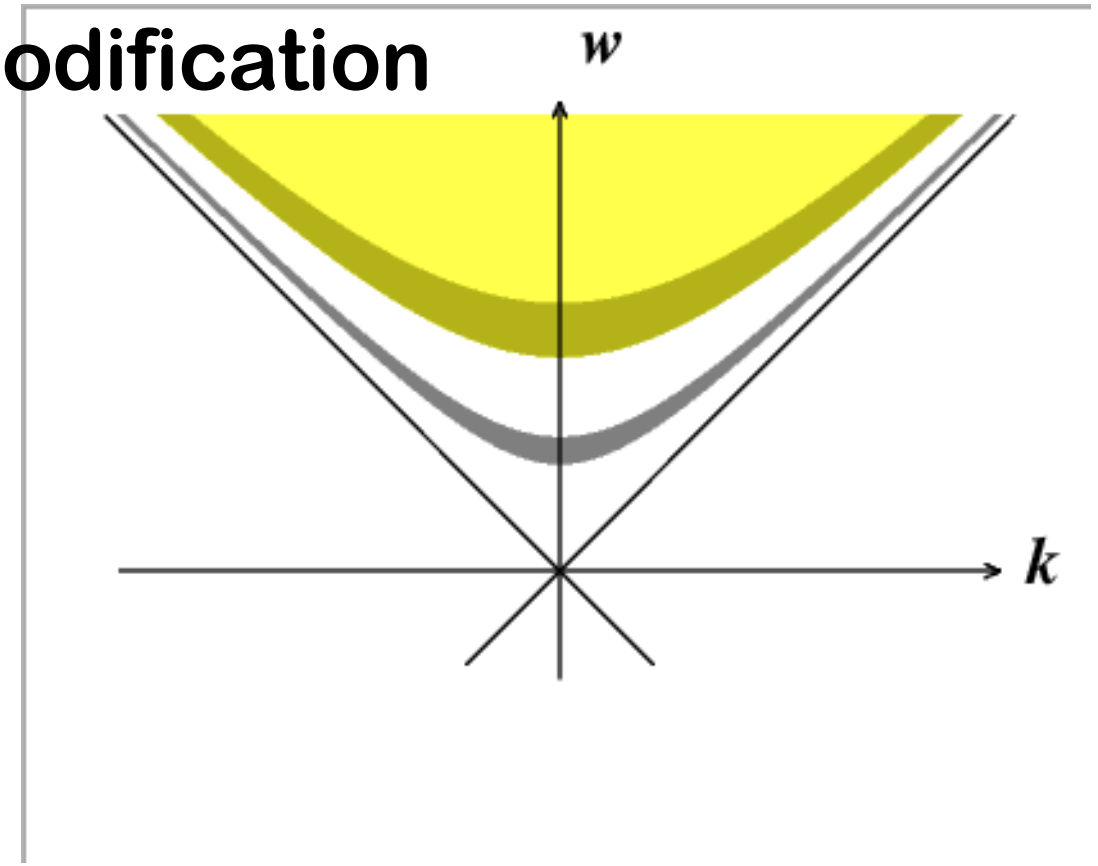
Region of spectrum

- vacuum



Region of spectrum

- mass modification



Region of spectrum

- medium fluctuation