Implications of the Dimension Two Gluon Condensate on the Deconfined Phase of QCD

Eugenio Megías[1], E. Ruiz Arriola[2] and L.L. Salcedo[2]

[1]Nuclear Theory Group, Physics Department, Brookhaven National Laboratory, Upton, New York 11973 USA.
[2]Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada, Spain.

Hot Quarks 2008
August 19th 2008, Estes Park, Colorado (USA).

Some References: E. Megías et al. JHEP 0601 (2006), PRD75 (2007), paper in preparation (2008).

Issues

Issues

Issues

1. Motivation
2. QCD and Trace Anomaly
   - Trace Anomaly
   - Power temperature corrections
   - Scale invariance and confinement
3. Dimension Two Gluon Condensate
   - Polyakov loop and dimension two gluon condensate
   - Power temperature corrections
   - Non Perturbative model
   - Non Perturbative contributions in the Free Energy
4. Dimension two gluon condensate and Trace Anomaly
   - Non perturbative contribution to the Trace Anomaly
   - General Formalism
5. Conclusions

Issues

Issues

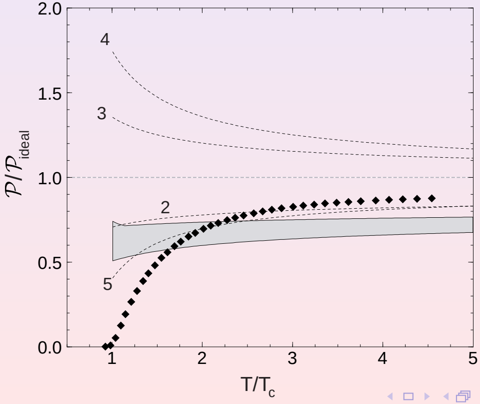
1. Motivation
2. QCD and Trace Anomaly
   - Trace Anomaly
   - Power temperature corrections
   - Scale invariance and confinement
3. Dimension Two Gluon Condensate
   - Polyakov loop and dimension two gluon condensate
   - Power temperature corrections
   - Non Perturbative model
   - Non Perturbative contributions in the Free Energy
4. Dimension two gluon condensate and Trace Anomaly
   - Non perturbative contribution to the Trace Anomaly
   - General Formalism
5. Conclusions

Motivation

Pressure of Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

E. Braaten and A. Nieto (1996), J.O. Andersen et al (1999).

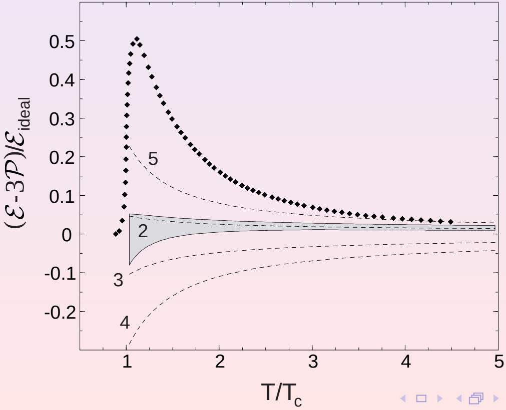


Motivation

Interaction measure in Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

E. Braaten and A. Nieto (1996), J.O. Andersen et al (1999).



Issues

Trace Anomaly

QCD Lagrangian:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \overline{q}^a_f (i\gamma_\mu D_\mu - m_f) q^a_f \,;$$

In the limit of massless quarks ($m_f = 0$), it is Invariant under scale ($\mathbf{x} \longrightarrow \lambda\mathbf{x}$) and chiral **Left** $\leftrightarrow$ **Right** transformations.

The "Classical " scale invariance is broken by quantum effects. They introduce a mass scale $\Lambda_{\rm QCD}$. Under a scale transformation ($\mu \longrightarrow e^\sigma \mu$):

$$g \longrightarrow g + \sigma\beta(g) \qquad \mathcal{L}_{\rm QCD} \longrightarrow \mathcal{L}_{\rm QCD} + \sigma\beta(g)\frac{\partial}{\partial g}\mathcal{L}_{\rm QCD}\,.$$

The **scale anomaly** can be computed as the trace of the energy-momentum tensor, so it is also known as Trace Anomaly:

$$\theta^\mu_\mu = \beta(g)\frac{\partial}{\partial g}\mathcal{L}_{\rm QCD} = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^a_{\mu\nu}$$

At finite temperature, from the partition function of Gluodynamics Z:

$$-\frac{\partial \log Z}{\partial(1/4g_0^2)} = \frac{V}{T}\langle(\bar{G}^a_{\mu\nu})^2\rangle \,.$$

After renormalization and using standard thermodynamics relations:

$$\left(4 - T\frac{\partial}{\partial T}\right)\frac{-T}{V}\log Z = \frac{\beta(g)}{2g}\langle(G^a_{\mu\nu})^2\rangle = \langle\theta^\mu_\mu\rangle = \epsilon - 3p \,.$$

The trace anomaly is related to the interaction measure $\Delta \equiv (\epsilon - 3P)/T^4$. In PT up to two loops (J.I.Kapusta (1979)):

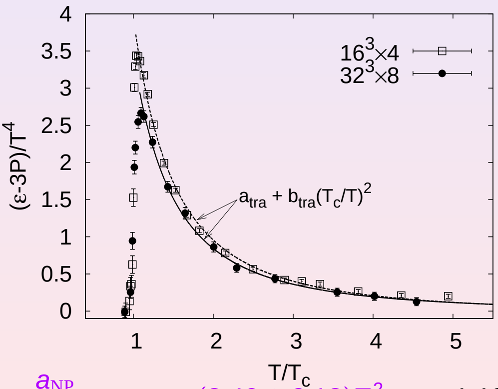
$$\frac{\epsilon - 3p}{T^4} = \frac{N_c(N_c^2 - 1)}{1152\pi^2}\beta_0 g(T)^4 + \mathcal{O}(g^5)$$

- Lattice data predicts a violent behaviour in powers of **T**. (Many groups: G. Boyd et al, NPB (1996), Y. Aoki et al (2006), ...).
- PT predicts a smooth dependence on T, because $g(T) \sim \log(T)$, so it is unable to reproduce this power behaviour, even if more and more orders are included (Andersen, Ann.Phys.317,2005).

Issues

Power temperature corrections from Lattice data

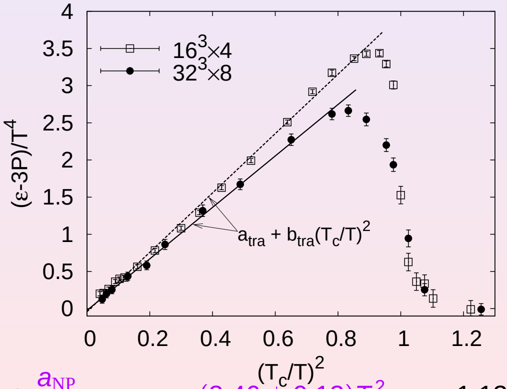
Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (3.46 \pm 0.13)T_c^2\,, \qquad 1.13T_c < T < 4.5T_c\,.$$

Power temperature corrections from Lattice data

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).



$$\frac{\epsilon - 3P}{T^4} = a_{\rm P} + \frac{a_{\rm NP}}{T^2}, \quad a_{\rm NP} = (3.46 \pm 0.13) T_c^2 , \qquad 1.13 T_c < T < 4.5 T_c .$$

Issues

Scale invariance and confinement

Consider a rectangular Wilson loop:

$$W(\mathcal{C}) = \exp\left(ig \int_{\mathcal{C}} A_\mu dx^\mu \right)$$

It is related to the potential $V_{q\bar{q}}(R)$ acting between charges q and $\bar{q}$:

$$W(\mathcal{C}) \to \exp\left(-T V_{q\bar{q}}(R)\right)$$

Scale transformations: $T \to \lambda T$, $R \to \lambda R$,
The only scale invariant solution is the Coulomb Potential:

$$V_{q\bar{q}} \sim \frac{1}{R}$$

Running coupling and string tension break scale invariance:

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_s(R)}{R} + \sigma R\,.$$

Issues

Polyakov loop and dimension two gluon condensate

E.Megías et al, JHEP 0601 (2006).

The vacuum expectation value of the Polyakov loop serves as an order parameter for the deconfinement phase transition in gluodynamics:

$$\mathbf{L} = \frac{1}{N_c} \langle \mathrm{tr}_c \Omega \rangle \equiv \frac{1}{N_c} \left\langle \mathrm{tr}_c \mathcal{P} \left(e^{ig \int_0^{1/T} dx_0 A_0(\vec{x}, x_0)} \right) \right\rangle .$$

$\mathcal{P}$ denotes path ordering. In the Polyakov gauge ($\partial_0 A_0(\vec{x}, x_0) = 0$) a gaussian approximation is possible:

$$\mathbf{L} = \frac{1}{N_c} \left\langle \mathrm{tr}_c \, e^{igA_{0,a}T_a/T} \right\rangle \longrightarrow \exp \left[-\frac{g^2 \langle A_{0,a}^2 \rangle}{4 N_c T^2} \right] .$$

- Cumulant expansion and vacuum saturation of condensates ($\langle A_0^{2k} \rangle = (2k-1)!! \langle A_0^2 \rangle^k +$ n.v.c.) are applied.
- Contribution from $\langle A_0^4 \rangle$ starts at $\mathcal{O}(g^6)$. So, it is valid up to $\mathcal{O}(g^5)$.

The dynamics of $A_0(\vec{x})$ can be described by the **dimensional reduced effective theory** of QCD (S.Nadkarni PRD27 (1983)):

$$\mathcal{L}'_{QCD} = -\frac{1}{T}\mathrm{tr}([D_i, A_0]^2) + \frac{m_D^2}{T}\mathrm{tr}(A_0^2) + \cdots$$

$D_{00}(\vec{k})\delta_{ab}$ is the propagator of the canonical fields $T^{-1/2}A_{0,a}(\vec{x})$. The integration of the propagator is related to the vacuum expectation value of the gluon fields (the dimension two gluon condensate):

$$\langle A_{0,a}^2 \rangle = (N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} D_{00}(\vec{k})\,.$$

Perturbative contribution (at leading order):

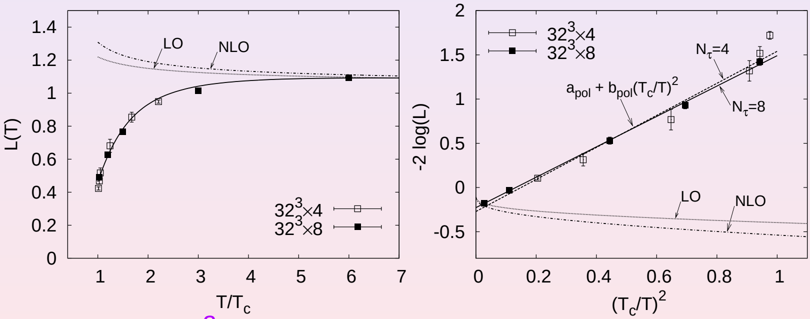
$$D_{00}^{\mathrm{P}}(\vec{k}) = \frac{1}{\vec{k}^2 + m_D^2}\,; \qquad \langle A_{0,a}^2 \rangle_{\mathrm{P}} = -\frac{(N_c^2 - 1)Tm_D}{4\pi} \sim T^2\,.$$

Leading order of E.Gava PLB105 (1981). It reproduces lattice data above $\sim 6T_D$. Below $6T_D$ non perturbative effects become important.

Issues

Power temperature corrections in the Polyakov loop

Renormalized Polyakov Loop $N_c = 3, N_f = 0$
O. Kaczmarek et al. PLB543 (2002).



$$-2\log(L) = a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,, \quad a_{\rm NP} = (1.81 \pm 0.13)T_c^2\,, \qquad 1.03T_c < T < 6T_c\,.$$

Perturbative result fails to reproduce lattice data in this regime.

Issues

Non Perturbative model

Consider **new phenomenological pieces** in the gluon propagator to take into account for non perturbative contributions (E.Megías JHEP0601(2006), see also K.G.Chetyrkin et al, NPB550 (1999)):

$$D_{00}(\vec{k}) = \underbrace{D_{00}^{\rm P}(\vec{k})}_{\sim 1/k^2} + \underbrace{D_{00}^{\rm NP}(\vec{k})}_{\sim 1/k^4}\,; \qquad \mathbf{D_{00}^{\rm NP}(k)} = \frac{\mathbf{m_G^2}}{(\mathbf{k^2} + \mathbf{m_D^2})^2}\,, \qquad \mathbf{m_G^2} > \mathbf{0}\,.$$

It produces a non perturbative contribution to the gluon condensate:

$$\langle A_{0,a}^2\rangle = \underbrace{\langle A_{0,a}^2\rangle_{\rm P}}_{\sim T^2} + \underbrace{\langle A_{0,a}^2\rangle_{\rm NP}}_{\sim T^0}\,; \qquad \langle \mathbf{A_{0,a}^2}\rangle_{\rm NP} = \frac{(\mathbf{N_c^2} - \mathbf{1})\mathbf{T}\mathbf{m_G^2}}{\mathbf{8}\pi\mathbf{m_D}} \sim \mathbf{T^0}\,.$$

Adding perturbative and non perturbative contributions:

$$-2\log \mathbf{L} = \frac{g^2\langle A_{0,a}^2\rangle_{\rm NP}}{2N_cT^2} = \underbrace{-\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi T}}_{\text{Pert.}\sim\log(T)} + \underbrace{\frac{g^2\langle A_{0,a}^2\rangle_{\rm NP}}{2N_cT^2}}_{\textbf{Non Pert.}\sim 1/T^2} \equiv a_{\rm P} + \frac{a_{\rm NP}}{T^2}\,.$$

Issues

Non Perturbative contributions in the Free Energy

Correlation functions of Polyakov loops define the free energy of a heavy $\overline{q}q$ pair (O.Kaczmarek et al, PLB543(2002)):

$$e^{-F_{q\bar{q}}(\vec{x},T)/T+c(T)} = \frac{1}{N_c^2}\langle \mathrm{tr}_c\Omega(\vec{x})\,\mathrm{tr}_c\Omega^\dagger(\vec{0})\rangle\,.$$

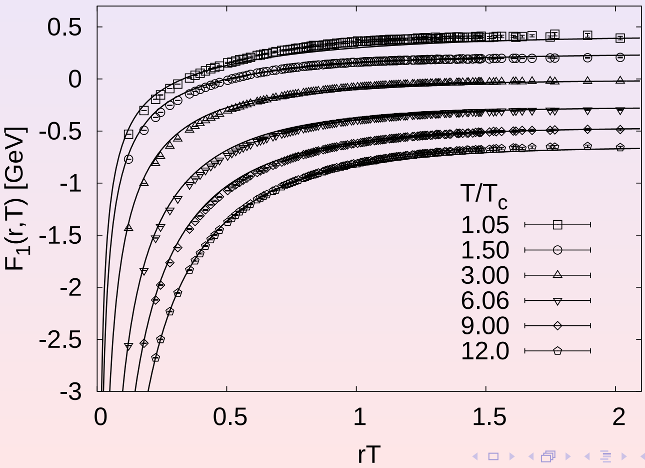
Pert. evaluation of Free Energy $\Rightarrow$ Expand Ω in powers of gA_0:

$$\langle A_{0,a}(\vec{x})A_{0,b}(\vec{y})\rangle = \delta_{ab}T\int\frac{d^3k}{(2\pi)^3}\,e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\,\underbrace{D_{00}(\vec{k})}_{D_{00}^{\mathrm{P}}+D_{00}^{\mathrm{NP}}}\,.$$

At leading order ($\mathcal{O}(g^2)$) and next to leading order ($\mathcal{O}(g^3)$):

$$\begin{aligned} F_1(r,T) \quad = \quad & -\frac{N_c^2-1}{2N_c}\left(\frac{g^2}{4\pi r}+\frac{1}{N_c^2-1}\frac{g^2\langle A_{0,a}^2\rangle_{\mathrm{NP}}}{T}\right)\mathbf{e}^{-\mathbf{m_D r}} \\ & -\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi}+\frac{1}{2N_c}\frac{g^2\langle A_{0,a}^2\rangle_{\mathrm{NP}}}{T}\,. \end{aligned}$$

Singlet Free Energy $N_c = 3, N_f = 0$
Lattice data (O. Kaczmarek PRD70 (2004)) vs NP model



Assyptotic limits

Taking the assyptotic limits:

- **$T \to 0$**: $F_1(r,T) \overset{T\to 0}{\sim} -\frac{N_c^2-1}{2N_c}\frac{g^2}{4\pi}\frac{1}{r} + \underbrace{\frac{g^3\langle A_{0,a}^2\rangle^{NP}}{2N_c}}_{\equiv\sigma} r \equiv V_{q\bar{q}}(r)\,.$

  $V_{q\bar{q}}(r)$ well known from lattice: S.Necco NPB622(2002).
- **$r \to \infty$**: $F_\infty(T) = F_1(r\to\infty, T) = -\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi} + \frac{g^2\langle A_{0,a}^2\rangle^{NP}}{2N_c T}\,.$

  $L(T) = e^{-F_\infty(T)/2T}$ also known from lattice: O.Kaczmarek.

From a fit of $V_{q\bar{q}}$ at $T=0$ ($F_1(r,T=0)$):

$$\sigma = (0.42(1)\,\mathrm{GeV})^2 \Longrightarrow g^2\langle A_{0,a}^2\rangle^{NP} = (0.82(2)\,\mathrm{GeV})^2\,.$$

From a fit of the Polyakov loop ($F_1(r=\infty,T)$):

$$a_{\mathrm{NP}} = (0.49(4)\,\mathrm{GeV})^2 \Longrightarrow g^2\langle A_{0,a}^2\rangle^{NP} = (0.84(4)\,\mathrm{GeV})^2\,.$$

These values agree with $\frac{1}{4}g^2\langle A_{\mu,a}^2\rangle_{T=0} = (0.8-1.8\,\mathrm{GeV})^2\,.$

Issues

Non perturbative contribution to the Trace Anomaly

Our model assumes the leading NP contribution to be encoded in the $A_{0,a}$ field. Taking $A_{i,a} = 0$:

$$\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle^{NP} = 2 \langle \partial_i A_{0,a} \partial_i A_{0,a} \rangle^{NP} = -6 m_D^2 \langle A_{0,a}^2 \rangle^{NP} \sim T^2 \, .$$

It reproduces the thermal behaviour of the trace anomaly:

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G^a_{\mu\nu})^2 \rangle = \underbrace{(\text{Pert.})}_{\sim T^4} - \underbrace{3g^2 \langle A_{0,a}^2 \rangle^{NP} \frac{\beta(g)}{g} T^2}_{\sim T^2} \, .$$

Values of the dimension two gluon condensate from a fit of:

Observable	$\mathbf{g^2 \langle A_{0,a}^2 \rangle_{NP}}$
Polyakov loop	$(3.22 \pm 0.07 T_c)^2$
Heavy $\overline{q}q$ free energy	$(3.33 \pm 0.19 T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24 T_c)^2$

Issues

General Formalism

The non-perturbative model can be applied to any thermal observable. Let Θ be an operator g-independent. The expectation value is:

$$\langle \Theta \rangle = \frac{Z_\Theta}{Z} , \qquad Z_\Theta = \int \prod_{\mu,a} \mathcal{D}A_{\mu,a} \, e^{-\int d^4x \, \mathcal{L}_{\mathrm{QCD}}(x)} \, \Theta$$

From Renormalization Group requirements

$$\left(\gamma_\Theta + T \frac{\partial}{\partial T} - r \frac{\partial}{\partial r} \right) \log\langle \Theta \rangle = -\Lambda_{\mathrm{QCD}} \frac{\partial}{\partial \Lambda_{\mathrm{QCD}}} \log\langle \Theta \rangle ,$$

where γ_Θ is the anomalous dimension. Because Λ_{QCD} is the only scale of QCD, the tachyonic gluon mass should depend on it. We take

$$m_G^2 = f(g) \Lambda_{QCD}^2 .$$

The final result is:

$$\left(\gamma_\Theta + \sum_i \mu_i \frac{\partial}{\partial \mu_i}\right) \log\langle\Theta\rangle = \frac{\beta(g)}{2g} \int d^4x \, \left(\langle (G^a_{\mu\nu})^2\rangle_\Theta - \langle (G^a_{\mu\nu})^2\rangle\right)$$
$$- \left(1 + \frac{\beta(g)}{2}\left(\frac{2}{g} - \frac{f'(g)}{f(g)}\right)\right) m_G^2 \int d^4x \, \left(\langle A^2_{0,a}\rangle_\Theta - \langle A^2_{0,a}\rangle\right) \, .$$

It imposes a constraint relation:

$$\left(1 + \frac{\beta(g)}{2}\left(\frac{2}{g} - \frac{f'(g)}{f(g)}\right)\right) = 0 \qquad \text{equivalent to} \quad \frac{\partial}{\partial \Lambda_{QCD}}\left(\frac{m_G^2}{g^2}\right) = 0$$

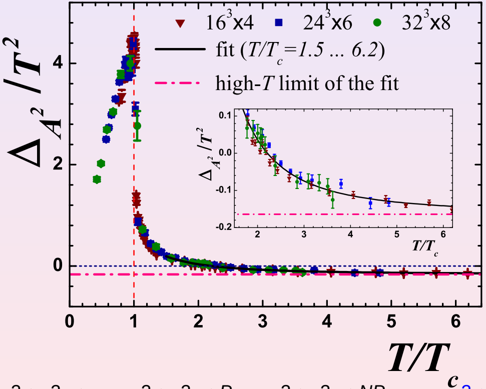
The result is $\beta_{\rm NP}(\mathbf{g}) \sim$ **g in the NP regime** $\Longrightarrow \alpha_{\rm NP}(\mathbf{T}) \sim \mathbf{1/T^2}$

(similar to Analytic PT: $\alpha(\mu) = \alpha_{\rm pert}(\mu) + \frac{4\pi}{\beta_0}\frac{\Lambda^2_{QCD}}{\Lambda^2_{QCD} - \mu^2}$). Comments:

- m_G signals an explicit breaking of scale invariance.
- This formula reproduces previous formulas for:
  - Polyakov loop: $\Theta = \frac{1}{N_c}\mathrm{tr}_c \Omega(\mathbf{x})$.
  - Heavy $\overline{q}q$ free energy: $\Theta = \frac{1}{N_c}\mathrm{tr}_c \, \Omega(\mathbf{x}) \frac{1}{N_c}\mathrm{tr}_c \, \Omega^\dagger(\mathbf{y})$.
  - Pressure and Trace Anomaly: $\epsilon - 3p \quad = \quad \frac{\beta(g)}{2g}\langle (G^a_{\mu\nu})^2\rangle$

Further checks of the model

Gluon asymmetry $N_c = 2, N_f = 0$
Lattice data (M. Chernodub (2008), arXiv:0805.3714)



$$\Delta_{A^2} = g^2\langle A^2_{0,a}\rangle - \frac{1}{3}\sum_i g^2\langle A^2_{i,a}\rangle$$

$$\Delta_{A^2}/T^2 \sim 1/T^2$$

$$g^2\langle A^2_{0,a}\rangle = \underbrace{g^2\langle A^2_{0,a}\rangle^P}_{\sim T^2} + \underbrace{g^2\langle A^2_{0,a}\rangle^{NP}}_{\sim \text{cte}} \Longrightarrow g^2\langle A^2_{0,a}\rangle/T^2 = a_p + a_{NP}/T^2 .$$

Conclusions:

- Trace anomaly, like other thermal observables in QCD (Polyakov loop, heavy $\overline{q}q$ free energy, pressure, energy density, entropy density), has a non perturbative behaviour near and above T_c characterized by power corrections in T.
- We propose a simple model to describe this behaviour. Non perturbative contributions come from the dimension two gluon condensate $\langle A_0^2 \rangle_{\rm NP}$.
- Renormalization Group arguments help to extend the model to any thermal observable.
- $\langle A_0^2 \rangle_{\rm NP}$ can be chosen to fit thermal observables. Its value agrees for all of them, and it is remarkably close to existing studies at $T = 0$. This result seems to imply an unified and coherent description of thermal observables in terms of the dimension two gluon condensate.