ChPT, transport coefficients, and resonances

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Chiral Perturbation Theory

- For low energies, $\lesssim 1\ \mathrm{GeV}$, and low temperatures, $\lesssim 300\ \mathrm{MeV}$, we are in the non-perturbative regime of QCD. But in this regime, the chiral symmetry of QCD is spontaneously broken:

$$\boxed{\chi \equiv \mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \equiv \mathrm{SU}(3)_{\mathrm{V}} \times \mathrm{SU}(3)_{\mathrm{A}} \longrightarrow \mathrm{SU}(3)_{\mathrm{V}}\ .}$$

- There, the degrees of freedom are the corresponding Goldstone bosons: pions, kaons and eta.

- Chiral symmetry is non-linearly realized on the Goldstone bosons: $U(x) \overset{\chi}{\mapsto} R\,U(x)\,L^{\dagger}$

$\in \mathrm{SU}(3)_{\mathrm{L}}$

Gell-Mann matrices

$$\text{with} \quad U(x) \equiv \exp\left(\mathrm{i}\frac{\phi(x)}{F_0}\right) \ , \quad \text{and} \quad \phi(x) = \sum_{a=1}^{8} \lambda_a \phi_a(x)$$

Goldstone bosons

$\in \mathrm{SU}(3)_{\mathrm{R}}$

a non-linear function

$$\Rightarrow \quad \boxed{[Q_a^{\mathrm{V}}, \phi_b] = \mathrm{i} f_{abc}\phi_c \ , \quad [Q_a^{\mathrm{A}}, \phi_b] = g_{ab}(\phi)\ .}$$

- **ChPT lagrangian:** The most general expansion in terms of derivatives of the $U(x)$ field and masses which respects all the symmetries of QCD:

$$\mathcal{L}_{\mathrm{ChPT}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \qquad \text{(infinite \# of terms)}$$

Leading and next-to-leading order lagrangians

- Leading order:

$$\mathcal{L}_2 = \frac{F_0^2}{4}\,\mathrm{Tr}\{(\nabla_\mu U)(\nabla^\mu U)^\dagger\} + \frac{F_0^2}{4}\,\mathrm{Tr}\{\chi U^\dagger + U\chi^\dagger\}\ .$$

- Next-to-leading order:

$$\begin{aligned}
\mathcal{L}_4 = {} & L_1 \left(\mathrm{Tr}\{(\nabla_\mu U)(\nabla^\mu U)^\dagger\}\right)^2 + L_2\,\mathrm{Tr}\{(\nabla_\mu U)(\nabla_\nu U)^\dagger\}\,\mathrm{Tr}\{(\nabla^\mu U)(\nabla^\nu U)^\dagger\} \\
& + L_3\,\mathrm{Tr}\{(\nabla_\mu U)(\nabla^\mu U)^\dagger(\nabla_\nu U)(\nabla^\nu U)^\dagger\} + L_4\,\mathrm{Tr}\{(\nabla_\mu U)(\nabla^\mu U)^\dagger\}\,\mathrm{Tr}\{\chi U^\dagger + U\chi^\dagger\} \\
& + L_5\,\mathrm{Tr}\{(\nabla_\mu U)(\nabla^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)\} + L_6 \left(\mathrm{Tr}\{\chi U^\dagger + U\chi^\dagger\}\right)^2 \\
& + L_7 \left(\mathrm{Tr}\{\chi U^\dagger - U\chi^\dagger\}\right)^2 + L_8\,\mathrm{Tr}\{U\chi^\dagger U\chi^\dagger + \chi U^\dagger \chi U^\dagger\} \\
& - \mathrm{i}L_9\,\mathrm{Tr}\{f^R_{\mu\nu}(\nabla^\mu U)(\nabla^\nu U)^\dagger + f^L_{\mu\nu}(\nabla^\mu U)^\dagger(\nabla^\nu U)\} + L_{10}\{U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu}\} \\
& + H_1\,\mathrm{Tr}\{f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu}\} + H_2\,\mathrm{Tr}\{\chi\chi^\dagger\}\ .
\end{aligned}$$

where the energy and temperature-independent constants $F_0, B_0, L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}, H_1, H_2$ are experimentally determined.

Perturbation Theory: Weinberg's Theorem

- Dimension D of a Feynman diagram:

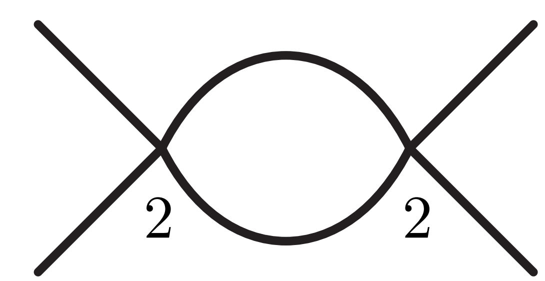
$$\text{Rescaling}: \begin{cases} p_i \mapsto t p_i \\ m_q \mapsto t^2 m_q \end{cases} \quad \Rightarrow \quad \mathcal{M}(tp_i, t^2 m_q) = t^{D} \mathcal{M}(p_i, m_q)\ .$$

amplitude of a diagram

- Weinberg's Theorem:

$$D = 2 + \sum_n N_n(n-2) + 2L\ .$$

Eg.,



$= \mathcal{O}(p^4)$

Number of vertices from $\mathcal{L}_n$

Number of loops

$p = E, |\boldsymbol{p}|, T, M$

Perturbation theory against the scales: $\Lambda_\chi \sim 1\ \mathrm{GeV}$ (for momenta), $\Lambda_T \sim 200\ \mathrm{MeV}$ (for temperatures).

Transport coefficients in hot gauge and scalar theories

[Jeon, PRD 52, 3591 (1995)]

[Arnold, Moore & Yaffe, JHEP 0011:011 (2000)]

- In Linear Response Theory (LRT), a transport coefficient is given by:

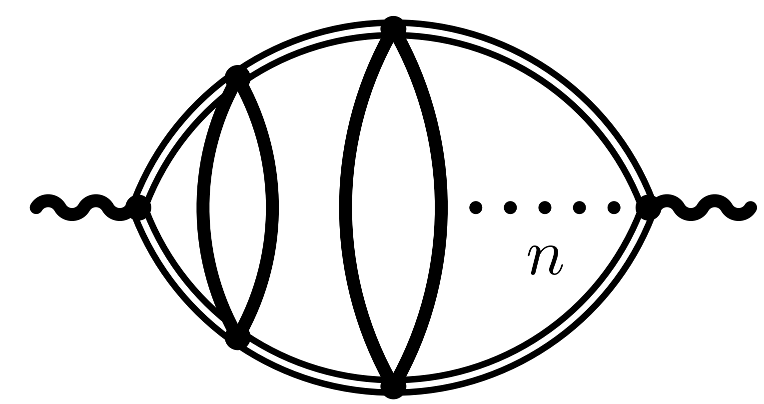
$$\mathcal{T} = C_{\mathcal{T}} \lim_{q^0 \to 0^+} \lim_{|\boldsymbol{q}| \to 0^+} \frac{\partial \rho_{\mathcal{T}}(q^0, \boldsymbol{q})}{\partial q^0} \ .$$

- For $T \gg m$, an infinite number of Feynman diagrams have to be summed, because of pinching poles:

$\lambda\phi^4$

$$G_{\mathrm{adv}}(p^0, \boldsymbol{p}) G_{\mathrm{ret}}(p^0, \boldsymbol{p}) \simeq \frac{\pi}{4E_{\boldsymbol{p}}^2 \Gamma_{\boldsymbol{p}}} \left[\delta(p^0 - E_{\boldsymbol{p}}) + \delta(p^0 + E_{\boldsymbol{p}}) \right] \ , \quad \text{and } \Gamma \sim \mathcal{O}(\lambda^2 T) \ .$$

only for lines which share the same momentum

- The most harmful diagrams are the ladder ones: 

$= \mathcal{O}(1/\lambda^2)$

- Bubble diagrams turn out to be subdominant: 

(but not for the bulk viscosity)

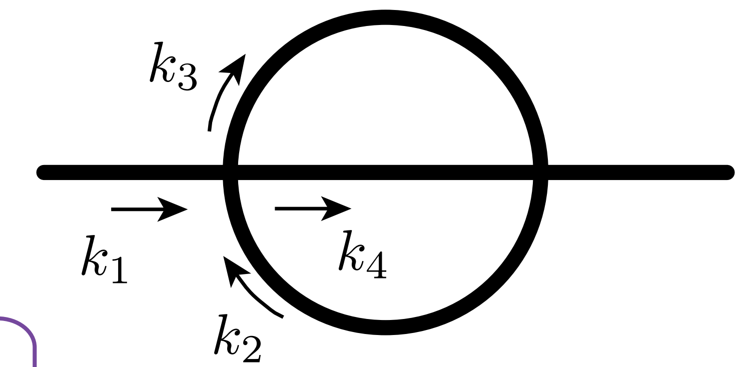
- Thus, resummation necessary in those theories. What happens in ChPT?.

Transport coefficients in ChPT

Particle width in ChPT

[Gomez Nicola & DFF, PRD 73, 045025 (2006)]

$$\Gamma(k_1) = \frac{1}{2}\int \frac{d^3\boldsymbol{k}_2}{(2\pi)^3}\, e^{-\beta E_2}\sigma_{\pi\pi} v_{\rm rel}(1-\boldsymbol{v}_1\cdot\boldsymbol{v}_2) \sim \mathrm{Im}$$



- Scattering cross section:

partial waves

$$\sigma_{\pi\pi}(s) \simeq \frac{32\pi}{3s}\left[|t_{00}(s)|^2 + 9|t_{11}(s)|^2 + 5|t_{20}(s)|^2\right] \ .$$

Unitarity and the Inverse Amplitude Method (IAM)

- ChPT violates the unitarity condition for high p: $\mathrm{S}^\dagger\mathrm{S}=1 \Rightarrow \mathrm{Im}\, t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2$, with $\sigma(s)\equiv\sqrt{1-4M_\pi^2/s}$.

  Because partial waves are essentially polynomials in p: $t_{IJ}(s) = t_{IJ}^{(1)}(s) + t_{IJ}^{(2)}(s) + \mathcal{O}(s^3)$.

- IAM:

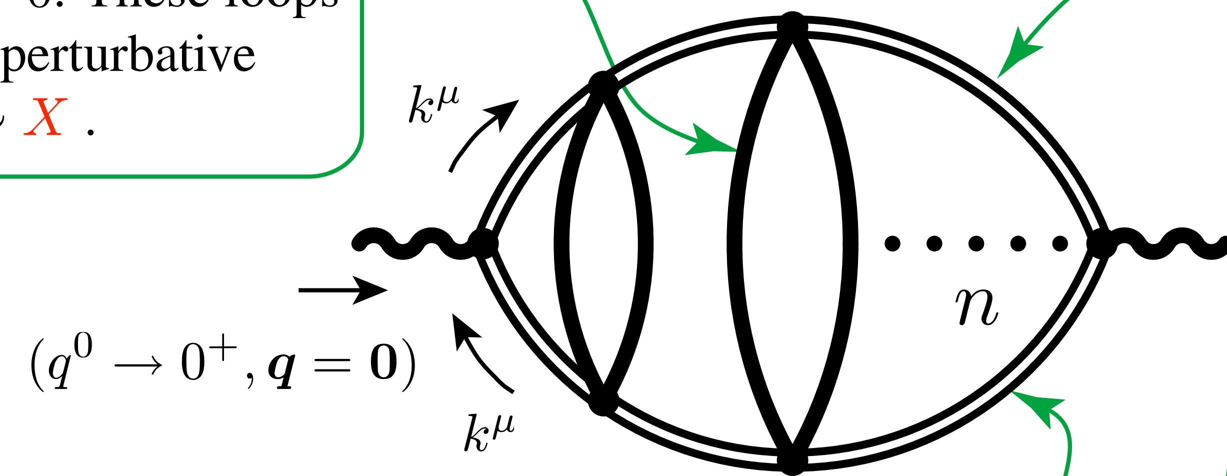
$$t_{IJ}(s) \simeq \frac{t_{IJ}^{(1)}(s)}{1-t_{IJ}^{(2)}(s)/t_{IJ}^{(1)}(s)} \ .$$

It verifies the unitarity condition exactly.

Diagramatic analysis

- Ladder diagrams:

lines with $\Gamma = 0$. These loops (rungs) give a perturbative contribution $\sim X$.



$$= \begin{cases} \mathcal{O}(X^n Y)\ , \\ \quad \text{for } T \ll M_\pi \\ \mathcal{O}(X^n Y^{n+1})\ , \\ \quad \text{for } T \simeq M_\pi \end{cases}$$

each pair of lines with $\Gamma \neq 0$ and equal momentum give a pinching pole contribution $\sim Y$

If $T \gtrsim M_\pi$, $X \sim 1$, and derivative vertices become important $\Rightarrow$ resummation may be relevant.

Eg., if we only consider *constant* vertices, for the DC conductivity:

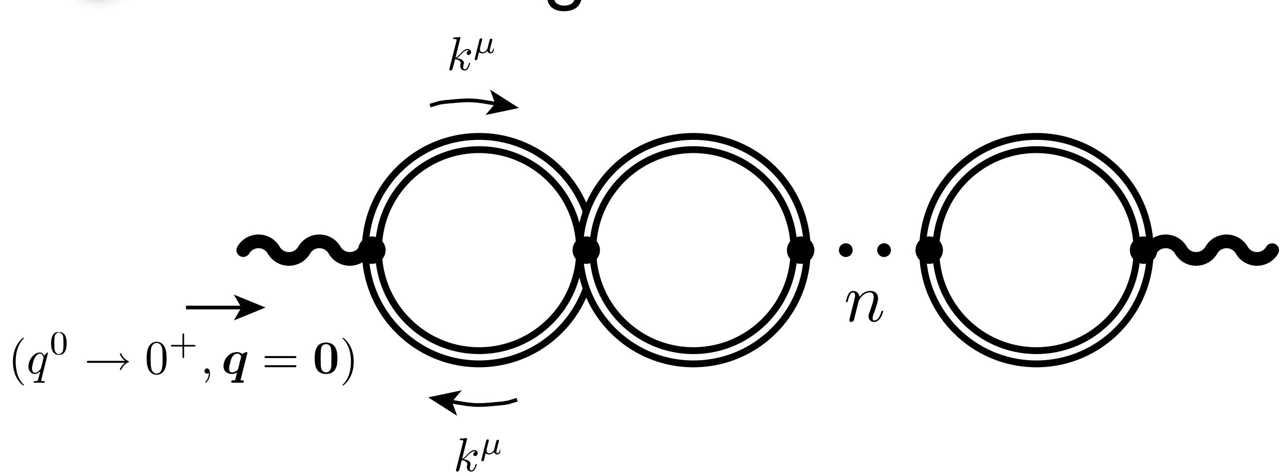
$$\text{For } T \ll M_\pi\ , \quad Y \sim \sqrt{\frac{M_\pi}{T}}\ , \quad X \sim \frac{1}{Y}\left(\frac{M_\pi}{4\pi F_\pi}\right)^2 .$$

$$\text{For } T \simeq M_\pi\ , \quad Y \sim 1\ , \quad X \lesssim \left(\frac{M_\pi}{4\pi F_\pi}\right)^2 .$$

pion decay constant

Weinberg's theorem does not give the correct order for TC at low T: $\mathcal{O}(p^{2n}) \gg \mathcal{O}(p^{4n})$.

- Bubble diagrams:



$$= \mathcal{O}(B^{(0)} X^{n-1})$$

contribution from a simple bubble $\sim Y$

This counting allows us to quickly obtain the functional form of TC at low T.

Electrical conductivity (pion gas)

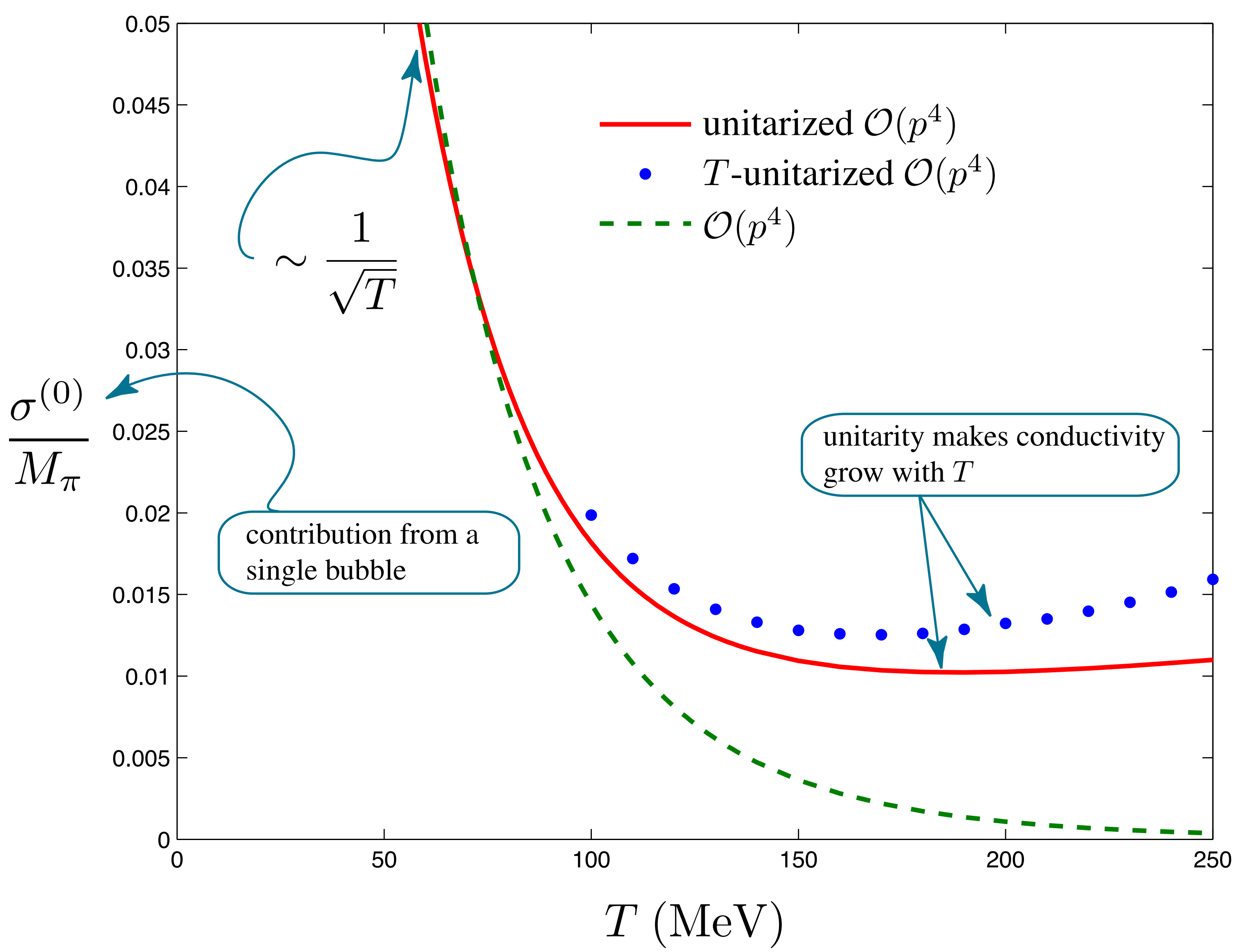
[Gomez Nicola & DFF, PRD 73, 045025 (2006)]

- Kubo formula:

electric current

$$\sigma = -\frac{1}{6}\lim_{q^0\to 0^+}\lim_{|\boldsymbol{q}|\to 0^+}\frac{\partial \rho_\sigma(q^0,\boldsymbol{q})}{\partial q^0}\ , \quad \rho_\sigma(q^0,\boldsymbol{q}) = 2\,\mathrm{Im}\,\mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}q\cdot x}\theta(t)\langle[J_i(x), J^i(0)]\rangle\ .$$

- Results:



According to Kinetic Theory (KT): $\sigma \sim \frac{e^2 n_{\mathrm{ch}}\tau}{M_\pi}$, but $\tau \sim 1/\Gamma$, and $\Gamma \sim n v \sigma_{\pi\pi}$.

For $T \ll M_\pi$, $n \sim (M_\pi T)^{3/2}\mathrm{e}^{-M_\pi/T}$, $v \sim \sqrt{T/M_\pi}$, and $\sigma_{\pi\pi}$ is a constant, $\Rightarrow$ $\sigma \sim 1/\sqrt{T}$. ✔

$$T \ll M_\pi : \quad \sigma^{(0)} \simeq 15\,\frac{e^2 F_\pi^4}{T^{1/2} M_\pi^{5/2}}$$

Application: soft-photon spectrum

- DC conductivity is related to the soft-photon spectrum emmited by the pion gas:

$$\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3\boldsymbol{q}} = \frac{1}{8\pi^3} n_{\mathrm{B}}(\omega) \rho^\mu{}_\mu(\omega = |\boldsymbol{q}|) \ , \quad \Rightarrow \quad \omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3\boldsymbol{q}}(\omega \to 0^+, \boldsymbol{q} = \boldsymbol{0}) = \frac{1}{4\pi^3} 3T\sigma(T) \ .$$

Ward identity: $q_\mu \rho^\mu{}_\nu = 0 \Rightarrow \rho_{00}(\omega \neq 0, \boldsymbol{q} = \boldsymbol{0}) = 0 \ .$

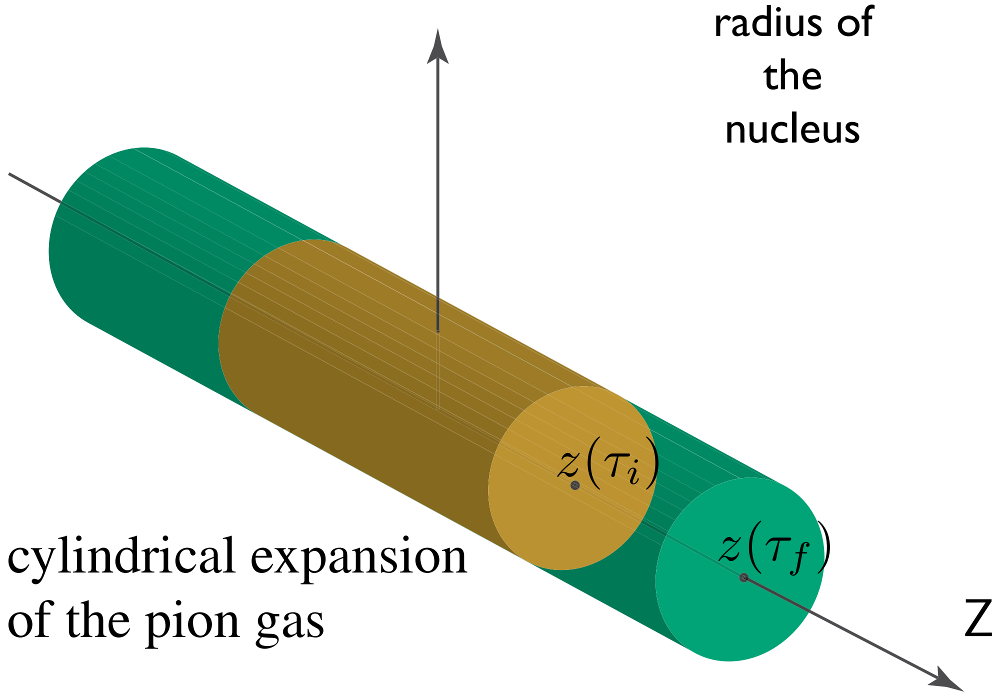
- Considering a Bjorken's hydrodynamical expansion:

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^3\boldsymbol{q}}(q_\perp \to 0) \simeq 2\pi R_A^2 \eta_{\mathrm{nucl}} \int_{\tau_{\mathrm{i}}}^{\tau_{\mathrm{f}}} \frac{3T(\tau)\sigma(T(\tau))}{4\pi^3} \tau \mathrm{d}\tau \ .$$

radius of the nucleus

rapidity of the colliding nuclei

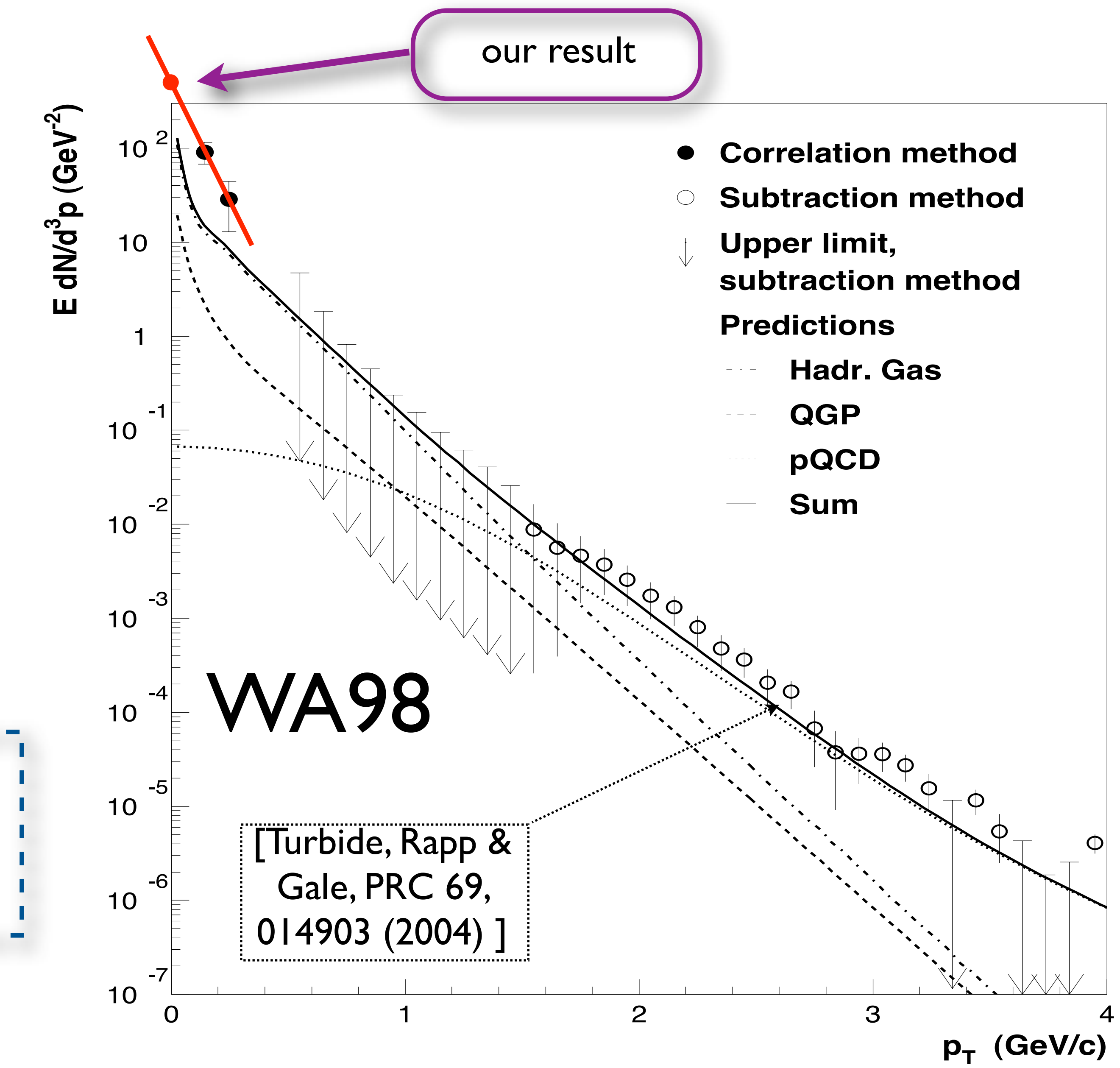
proper time



cylindrical expansion of the pion gas

$\Gamma \neq 0$ important for low energies (LPM effect)

[Rapp & Liu, NPA 796, 101 (2007)]



Thermal conductivity (pion gas)

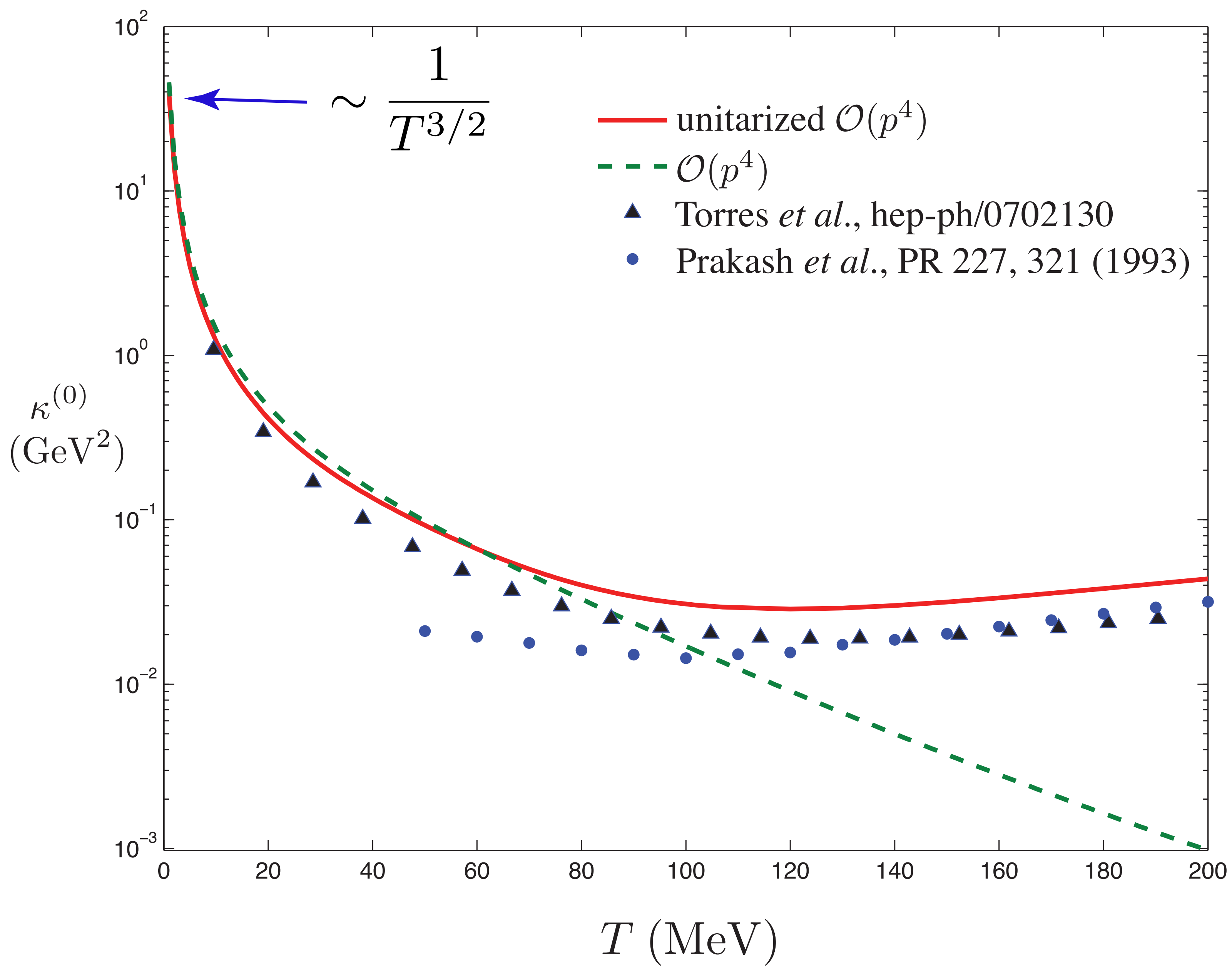
[Gomez Nicola & DFF, Int.J.Mod.Phys.E16:3010]

- Kubo formula:

energy-momentum tensor

$$\kappa = -\frac{\beta}{6} \lim_{q^0 \to 0^+} \lim_{|\boldsymbol{q}| \to 0^+} \frac{\partial \rho_\kappa(q^0, \boldsymbol{q})}{\partial q^0} \ , \qquad \rho_\kappa(q^0, \boldsymbol{q}) = 2 \, \mathrm{Im} \, \mathrm{i} \int \mathrm{d}^4 x \ \mathrm{e}^{\mathrm{i} q \cdot x} \theta(t) \langle [T_{0i}(x), T^{0i}(0)] \rangle \ .$$

- Results:



From KT: $\kappa \sim c_p l v$.

For $T \ll M_\pi$, $c_p \sim T^{-1/2} \mathrm{e}^{-M_\pi/T}$, $\Rightarrow$ $\kappa \sim T^{-3/2}$. ✔

$$T \ll M_\pi : \qquad \kappa^{(0)} \simeq 10 \frac{F_\pi^4}{T^{3/2} M_\pi^{1/2}}$$

Shear and bulk viscosities

- In presence of viscosities, the energy-momentum of the fluid is modified in the way:

velocity

$$T_{ij} = p\delta_{ij} - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right) - \zeta\delta_{ij}\nabla\cdot\boldsymbol{u}\ .$$

bulk viscosity

pressure

shear viscosity

- In LRT:

$$\eta = \frac{1}{20}\lim_{q^0\to 0^+}\lim_{|\boldsymbol{q}|\to 0^+}\frac{\partial\rho_\eta(q^0,\boldsymbol{q})}{\partial q^0}\ , \qquad \zeta = \frac{1}{2}\lim_{q^0\to 0^+}\lim_{|\boldsymbol{q}|\to 0^+}\frac{\partial\rho_\zeta(q^0,\boldsymbol{q})}{\partial q^0}\ ,$$

with

$$\rho_\eta(q^0,\boldsymbol{q}) = 2\,\mathrm{Im}\,\mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}q\cdot x}\theta(t)\langle[\pi_{ij}(x),\pi^{ij}(0)]\rangle\ ,$$

$$\rho_\zeta(q^0,\boldsymbol{q}) = 2\,\mathrm{Im}\,\mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}q\cdot x}\theta(t)\langle[\tilde{\mathcal{P}}(x),\tilde{\mathcal{P}}(0)]\rangle\ .$$

where

$= \text{energy density}$

$$\pi_{ij} \equiv T_{ij} - g_{ij}T^l{}_l/3\ , \qquad \tilde{\mathcal{P}} \equiv \overbrace{-T^l{}_l/3}^{=\ \text{pressure}} - v_s^2 T_{00}\ .$$

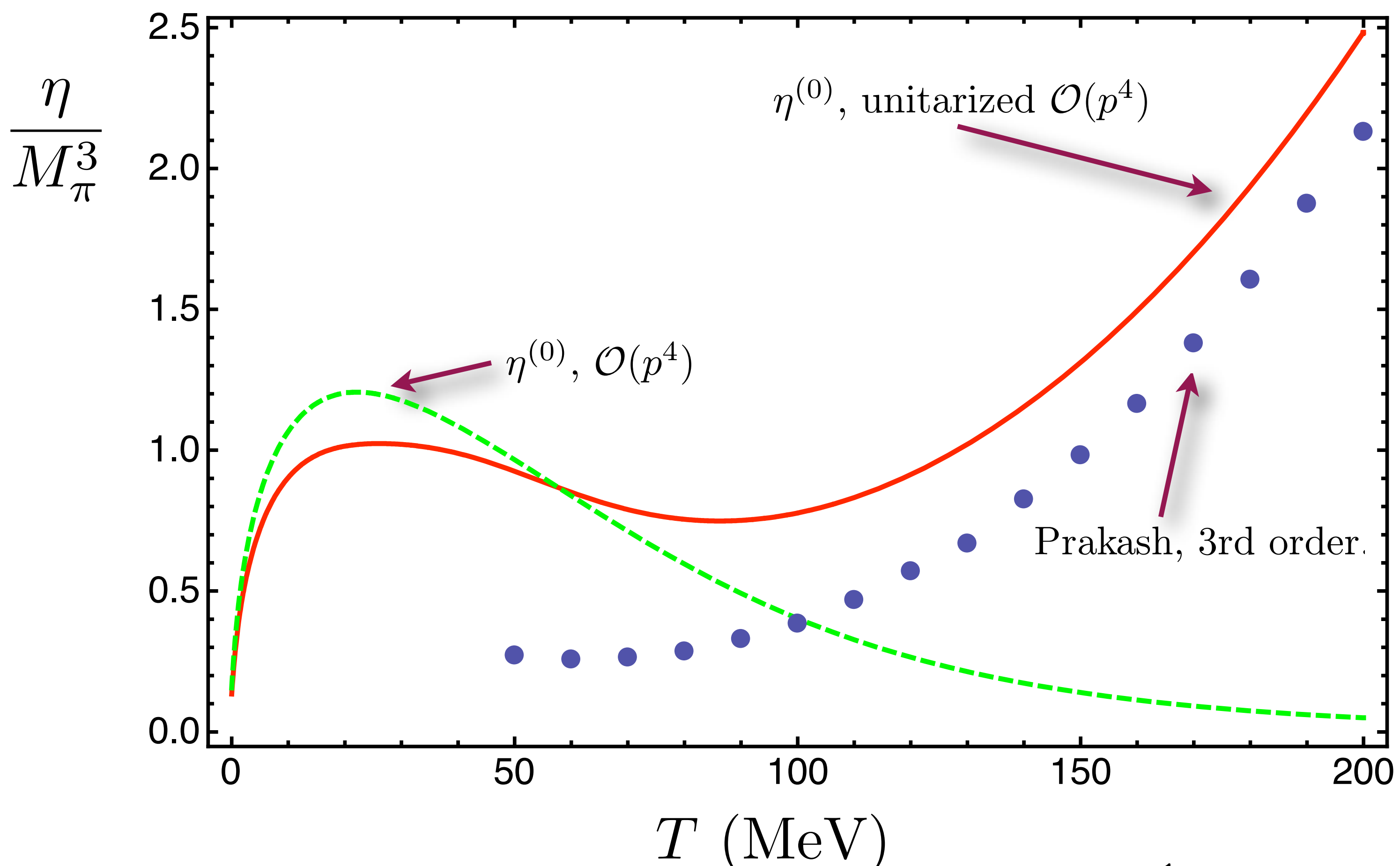
speed of sound in the fluid

$g_{\mu\nu} = \mathrm{diag}\,(+,-,-,-)$

Shear viscosity (pion gas)

[Gomez Nicola & DFF, Int.J.Mod.Phys.E16:3010]

- Results:



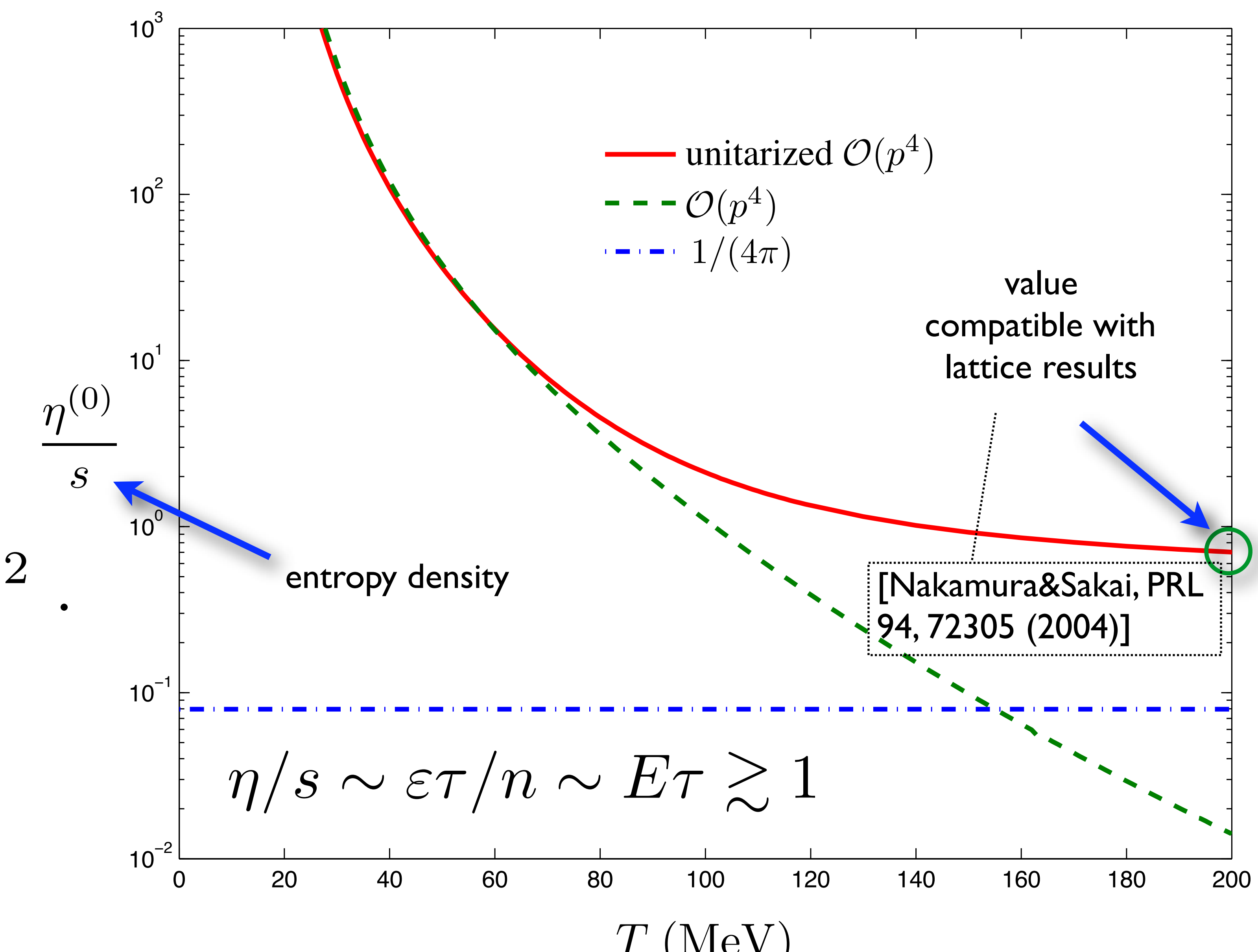
For η and ζ ladder diagrams could be important for $T < M_\pi$.

Good agreement with KT $\Rightarrow$ cancellation of ladders?

From KT: $\eta, \zeta \sim M_\pi v n l$, but $l \sim \dfrac{1}{\sigma_{\pi\pi} n}$.

So for $T \ll M_\pi$, $\eta, \zeta \sim \sqrt{T}$. ✔

- AdS/CFT bound: [Kovtun, Son & Starinets, 05]



- Sound attenuation length: $\omega = v_s k + \frac{1}{2} \mathrm{i}\Gamma_s k^2$.

  Neglecting bulk viscosity, $\Gamma_s \simeq \dfrac{4\eta}{3sT}$.

  At $T = 180$ MeV, we obtain $\Gamma_s \simeq 1.1$ fm.

[Teaney, PRC 68, 034913 (2003)]

Bulk viscosity and the trace anomaly

[Kharzeev & Tuchin, arXiv:0705.4280 [hep-ph]; Kharsch, Kharzeev & Tuchin, PLB 663, 217 (2008)]

- Trace anomaly of QCD: $\partial_\nu J^\nu_{\mathrm{dil}} = T^\mu{}_\mu = \dfrac{\beta(g)}{2g} G^a_{\mu\nu} G^{\mu\nu}_a + (1+\gamma(g))\bar{q}Mq$
- Bulk viscosity: $\zeta = \dfrac{1}{9} \lim_{\omega\to 0^+} \dfrac{1}{\omega} \int_0^\infty \mathrm{d}t \int \mathrm{d}^3\boldsymbol{x}\, \mathrm{e}^{\mathrm{i}\omega t} \langle [\hat{T}^\mu{}_\mu(x), \hat{T}^\nu{}_\nu(0)]\rangle = \dfrac{\pi}{9} \lim_{\omega\to 0^+} \dfrac{\rho(\omega,\mathbf{0})}{\omega}$
- Sum rule:

$$2\int_0^\infty \frac{\rho(u,\mathbf{0})}{u}\,\mathrm{d}u = Ts\left(\frac{1}{c_s^2}-3\right) - 4(\epsilon-3P) + \left(T\frac{\partial}{\partial T}-2\right)\langle m\bar{q}q\rangle^* - 4\langle T^\mu_\mu\rangle_0 + 6(M_\pi^2+F_\pi^2+M_K^2F_K^2)$$

$\langle\cdot\rangle^* \equiv \langle\cdot\rangle_T - \langle\cdot\rangle_0$

- Ansatz: $\dfrac{\rho(\omega,\mathbf{0})}{\omega} = \dfrac{9\zeta}{\pi}\dfrac{\omega_0^2}{\omega_0^2+\omega^2}$

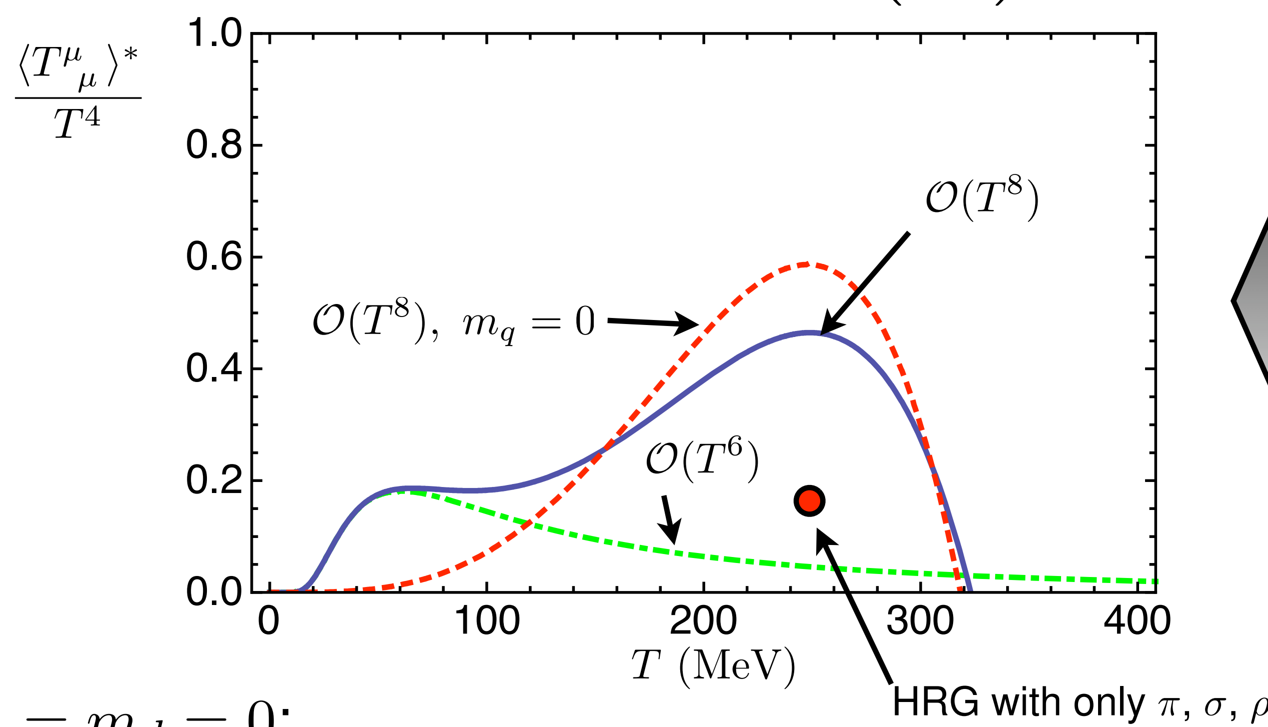
$$9\omega_0\zeta = Ts\left(\frac{1}{c_s^2}-3\right) - 4(\epsilon-3P) + \left(T\frac{\partial}{\partial T}-2\right)\langle m\bar{q}q\rangle^* - 4\langle T^\mu_\mu\rangle_0 + 6(M_\pi^2+F_\pi^2+M_K^2F_K^2)$$

$\langle T^\mu_\mu\rangle_T = \epsilon - 3P$

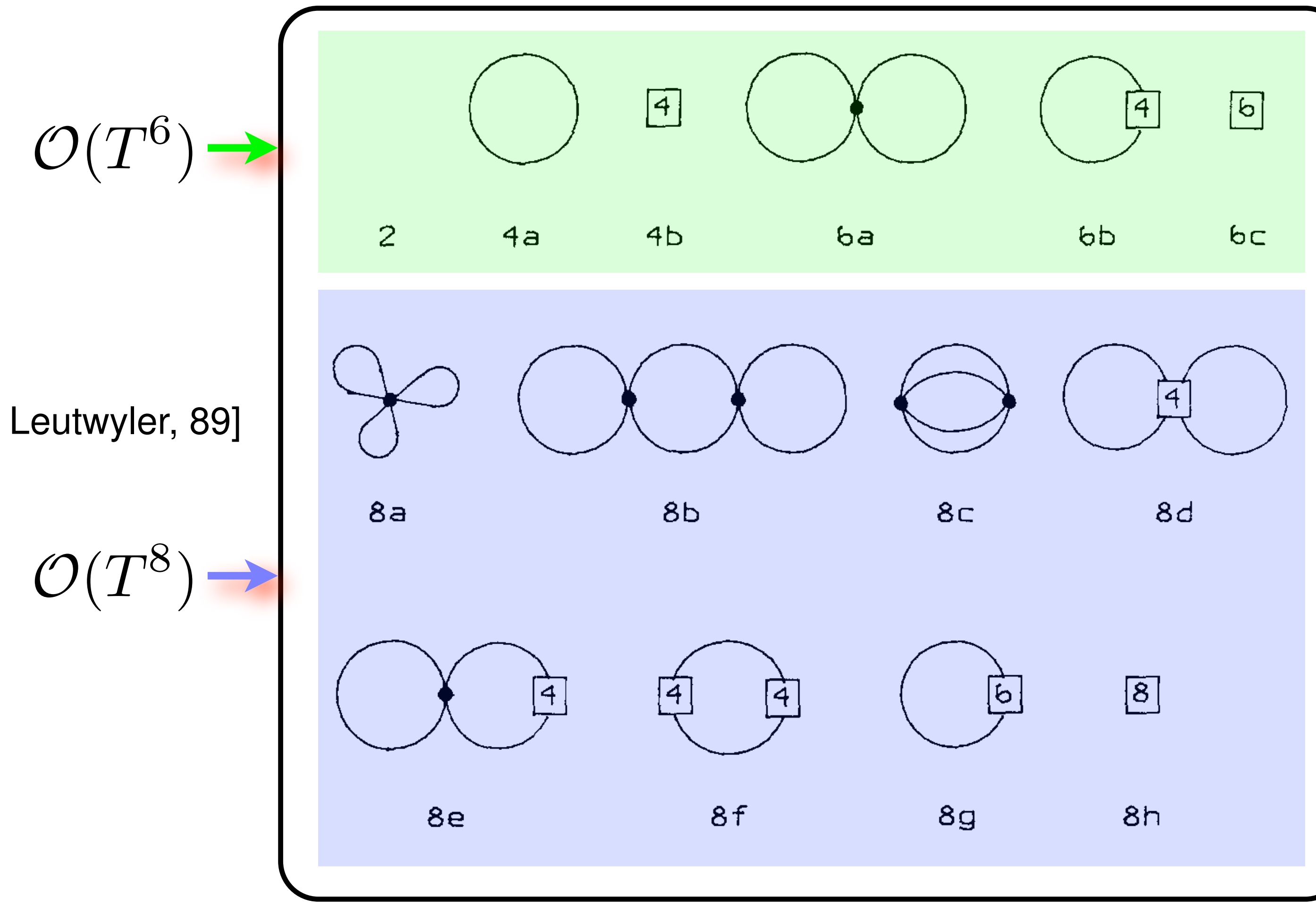
$\omega_0 \sim 1\ \mathrm{GeV}$

Trace anomaly for a pion gas (in preparation)

- From ChPT: $\langle T^{\mu}{}_{\mu} \rangle_T = T^5 \dfrac{\mathrm{d}}{\mathrm{d}T}\left(\dfrac{P}{T^4}\right)$



Pressure
[Gerber & Leutwyler, 89]



For $m_u = m_d = 0$:

$$\langle T^{\mu}{}_{\mu}\rangle^* = \frac{\pi^2}{270}\frac{T^8}{F_\pi^4}\left(\ln\frac{\Lambda_p}{T} - \frac{1}{4}\right) \;, \quad \Lambda_p \sim 400 \text{ MeV} \;.$$

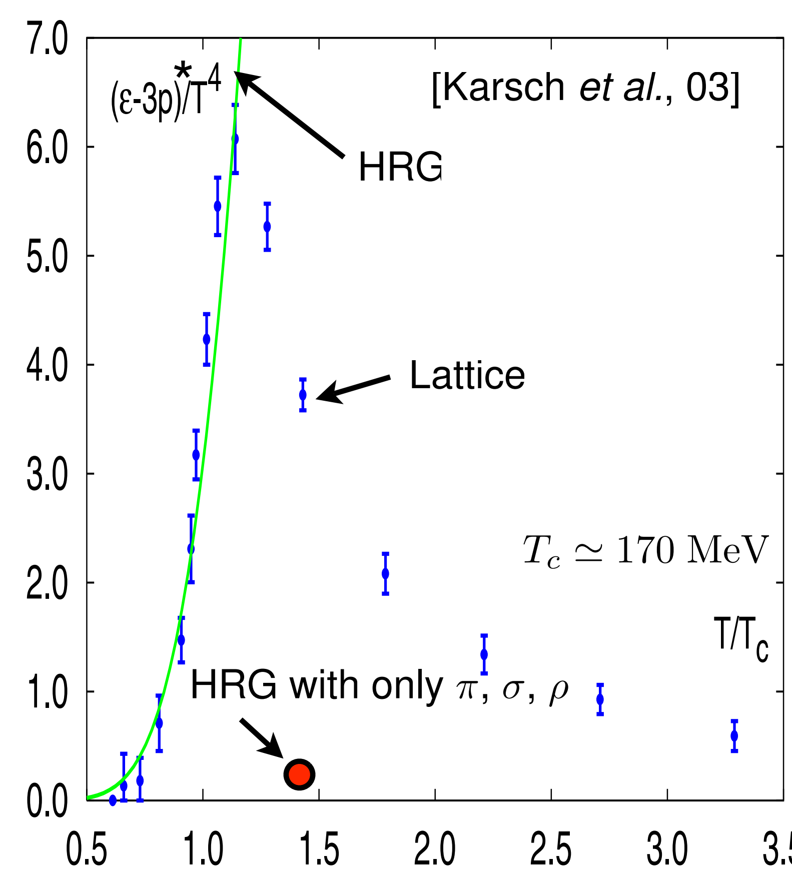
- Hadron Resonance Gas & Lattice (2+1 q):

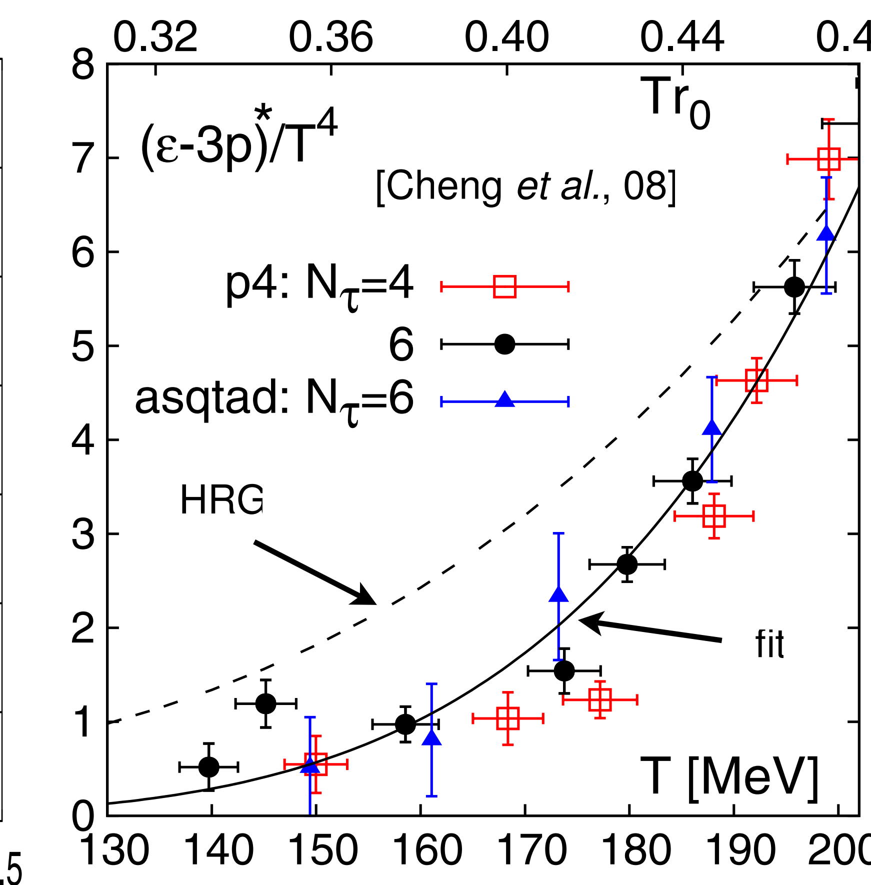
HRG approximation: all mesonic and baryonic resonances up to 2 GeV, 1026 in total, introduced as free states.

[Karsch et al., 03]

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = \sum_{i=1}^{1026} \frac{\epsilon_i - 3P_i}{T^4}$$

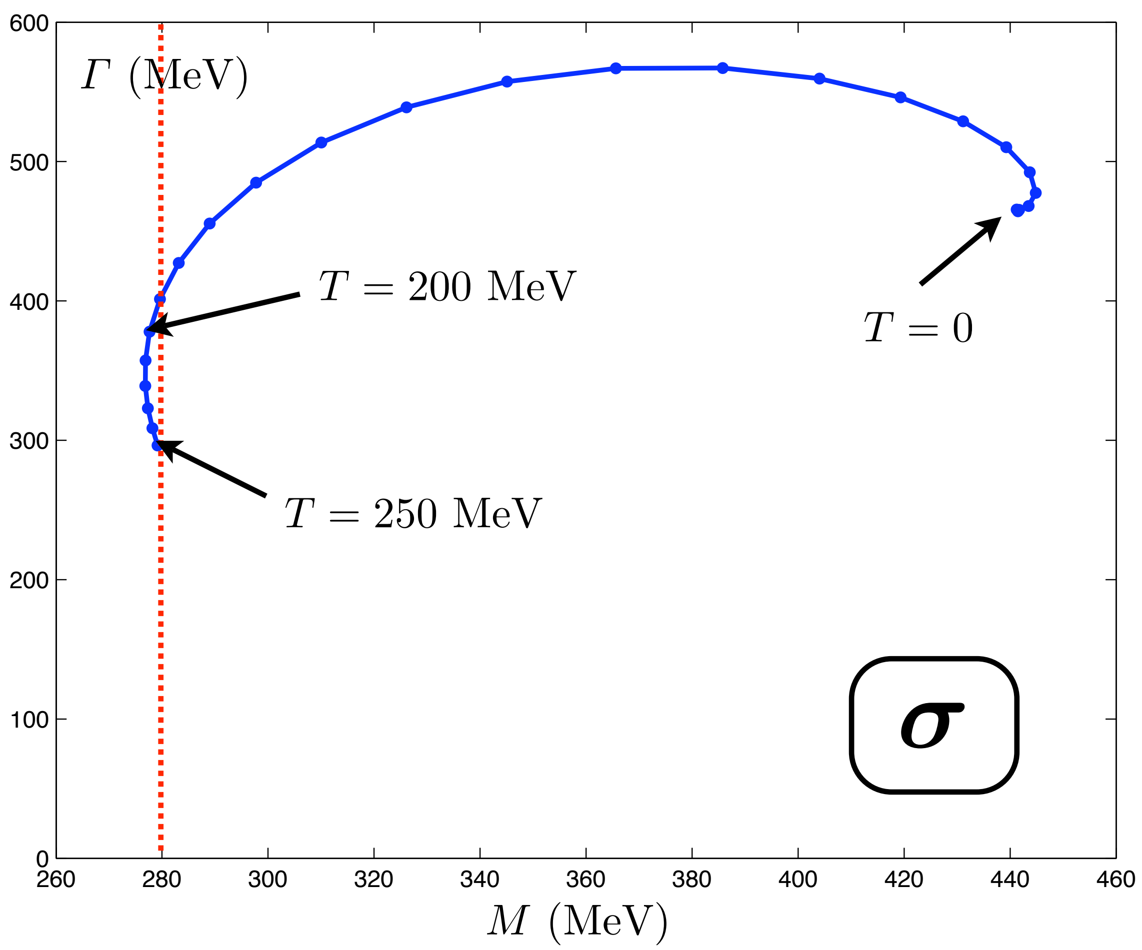
$$\stackrel{*}{=} \sum_{i=1}^{1026} \frac{g_i}{2\pi^2} \sum_{k=1}^{\infty} (-\eta)^{k+1} \frac{(\beta m_i)^3}{k} K_1(k\beta m_i)$$

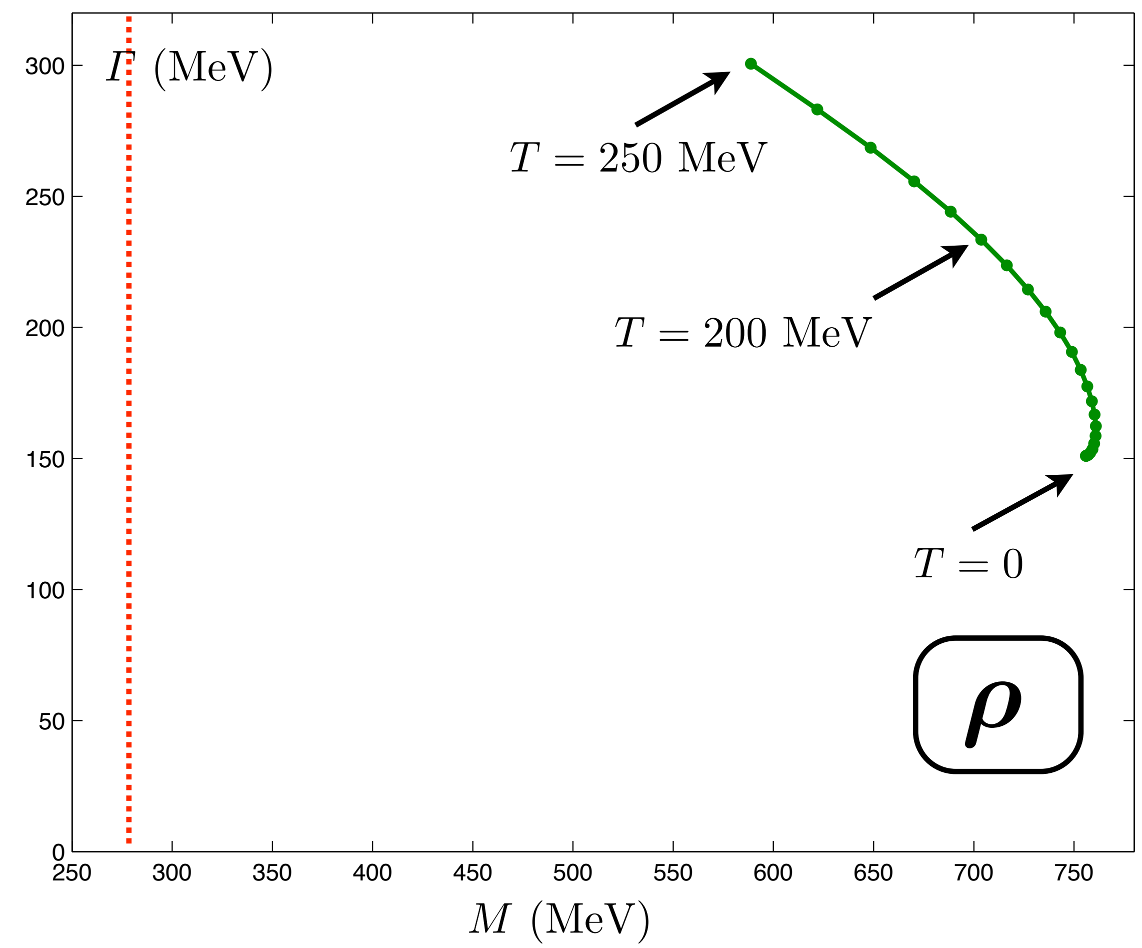




Behavior of the σ and ρ resonances in medium

- Finite temperature:





[Gomez Nicola, Herruzo & DFF, PRD 76, 085020 (2007)]

For an improved study of temperature and nuclear density effects for the σ using the Bethe-Salpeter eq. see

[Cabrera, Gomez Nicola, & DFF, about to send it to arXiv]

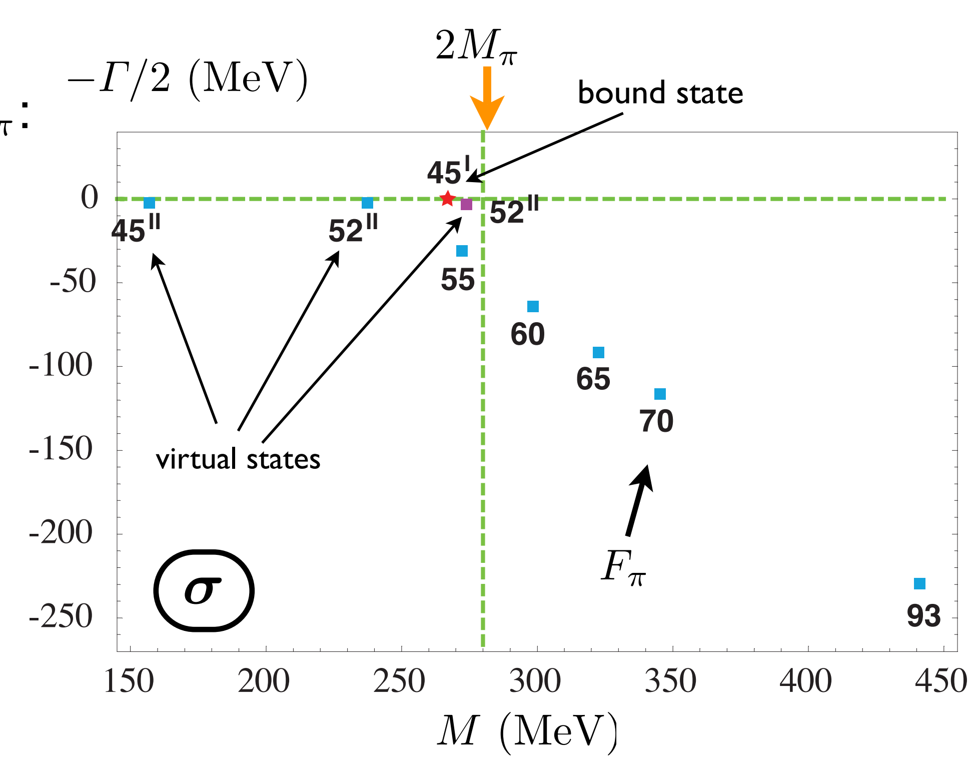
- Finite nuclear density:

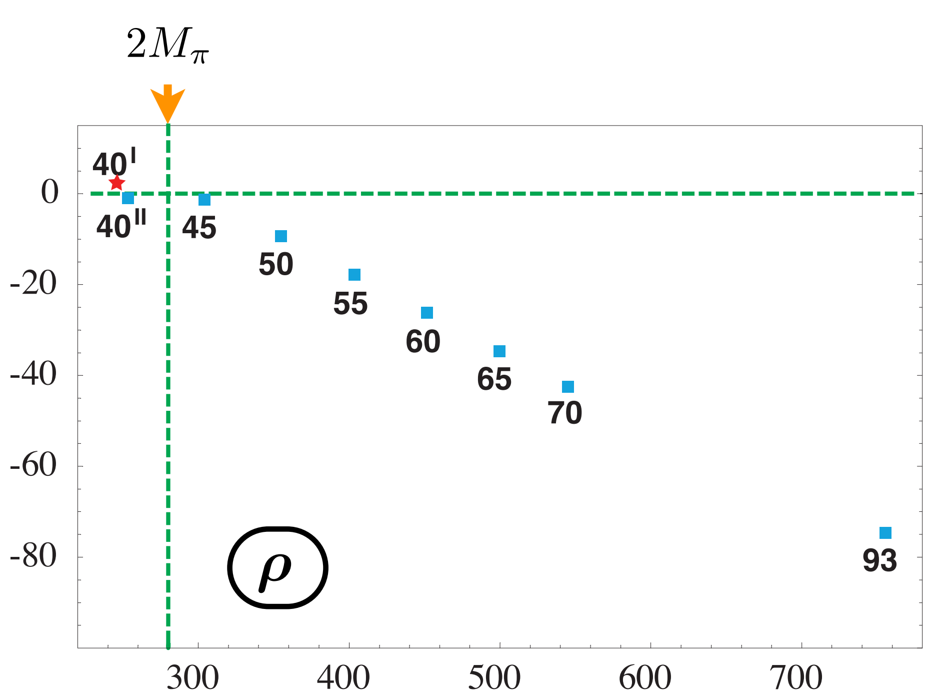
We can encode approximately nuclear density effects in F_π:

[Thorsson & Wirzba, NPA 589, 633 (1995)]

$$\frac{F_\pi^2(\rho)}{F_\pi^2} \simeq \frac{\langle\bar{q}q\rangle(\rho)}{\langle\bar{q}q\rangle(0)} \simeq \left(1-\frac{\sigma_{\pi N}}{M_\pi^2 F_\pi^2}\rho\right)+\mathcal{O}(M_\pi)$$
$$\simeq \left(1-0.35\frac{\rho}{\rho_0}\right)+\mathcal{O}(M_\pi)\ ,$$

where $\sigma_{\pi N} \simeq 45$ MeV, and $\rho_0 \simeq 0.17\ \mathrm{fm}^{-3}$.





The role of resonances in the trace anomaly and the bulk viscosity (in prep.)

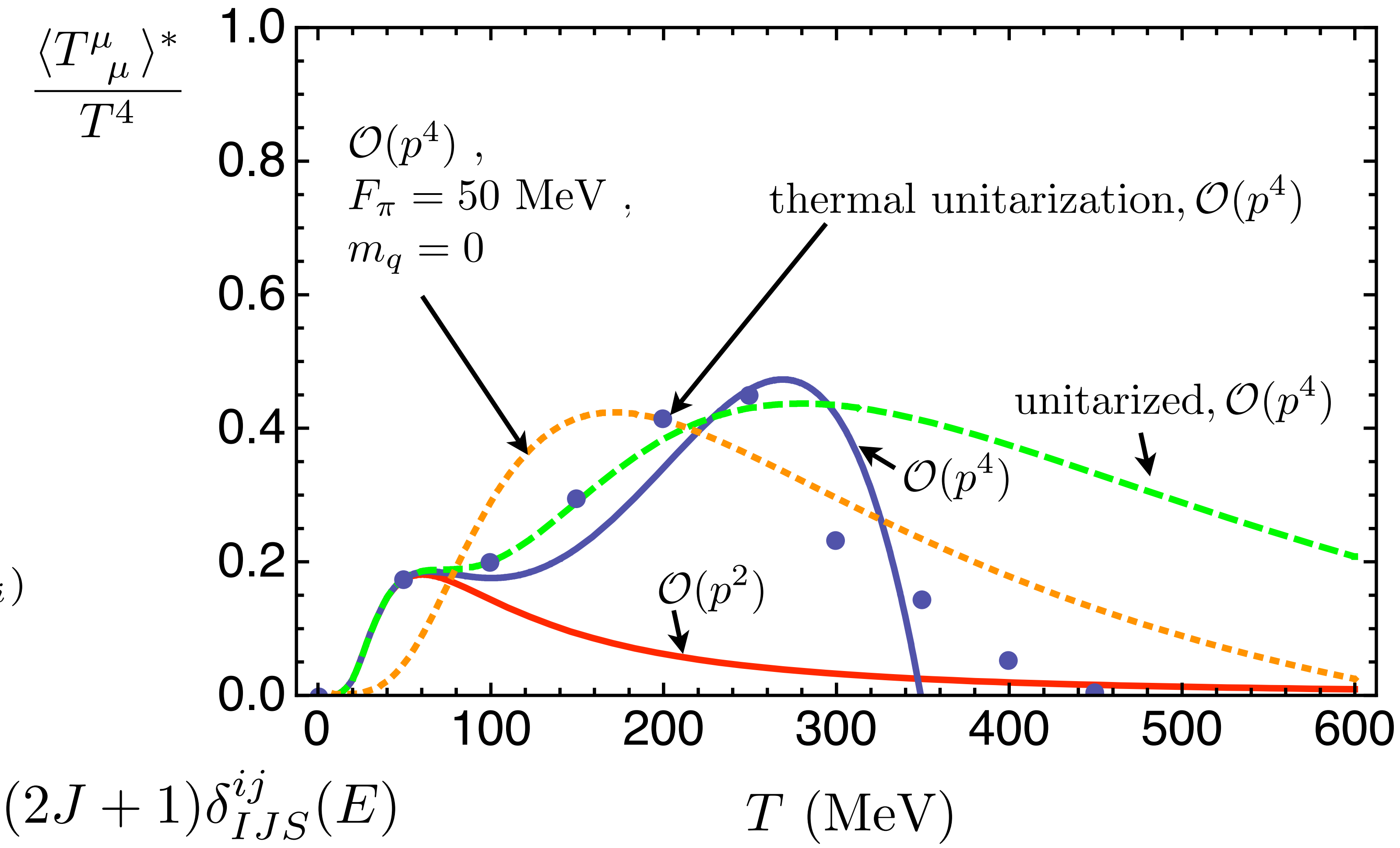
- Trace anomaly:

Virial Gas Approximation (dilute system):

$$\beta P = \sum_i \left(B_i^{(1)} \xi_i + B_i^{(2)} \xi_i^2 + \sum_{j \geq i} B_{\rm int} \xi_i \xi_j + \dots \right)$$

$$\xi_i \equiv \mathrm{e}^{\beta(\mu_i - m_i)} \ , \quad B_i^{(n)} = \frac{g_i \eta_i^{n+1}}{2\pi^2 n} \int_0^\infty \mathrm{d}p \ p^2 \mathrm{e}^{-n\beta(E_i - m_i)}$$

$$B_{ij}^{int} = \frac{\mathrm{e}^{\beta(m_i+m_j)}}{2\pi^3} \int_{m_i+m_j}^{\infty} \mathrm{d}E \ E^2 K_1(\beta E) \sum_{I,J,S} (2I+1)(2J+1) \delta_{IJS}^{ij}(E)$$

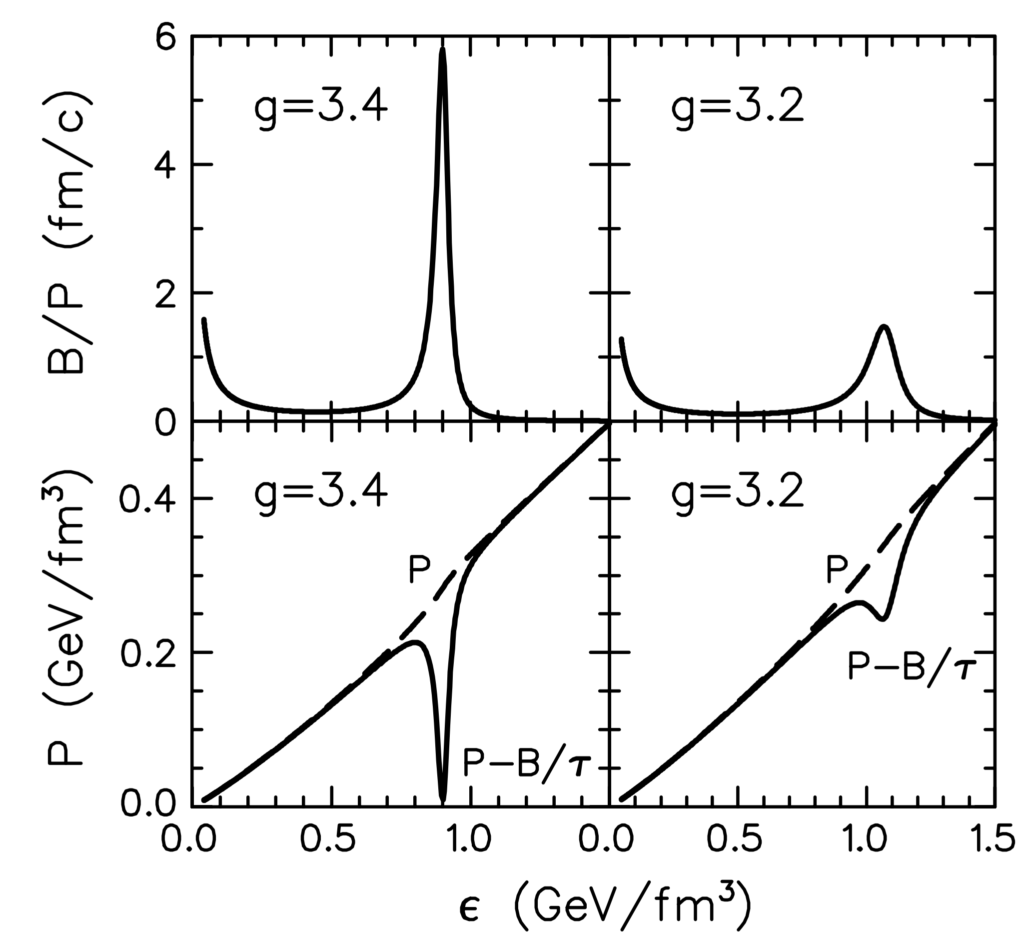


Therefore, according to the sum rule, we do not expect a big effect in the bulk viscosity from in-medium resonances.

- Bulk viscosity in the linear sigma model:

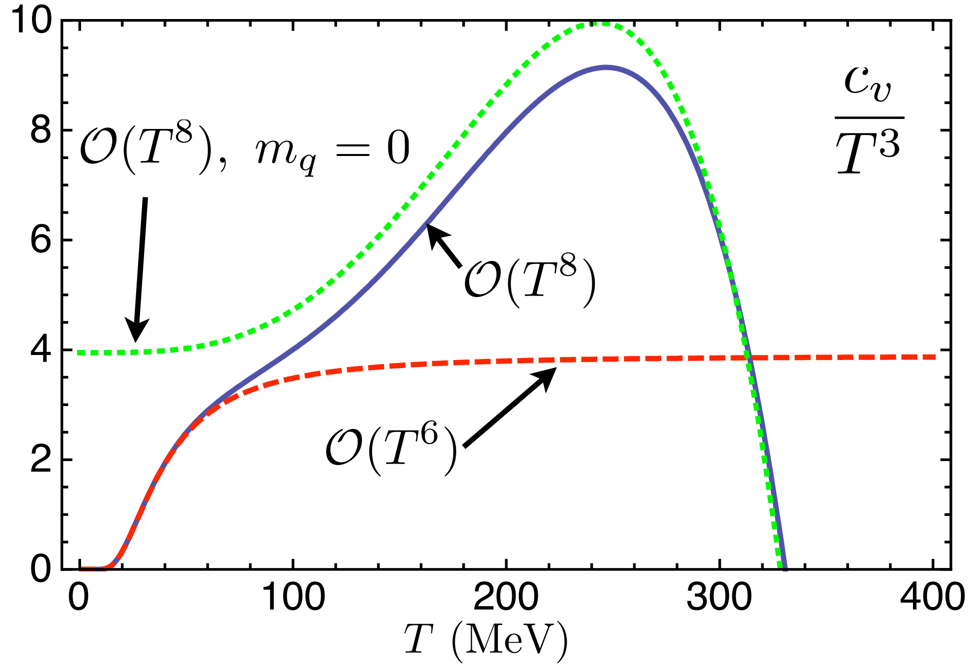
[Paech & Pratt, PRC 74, 014901 (2006)]

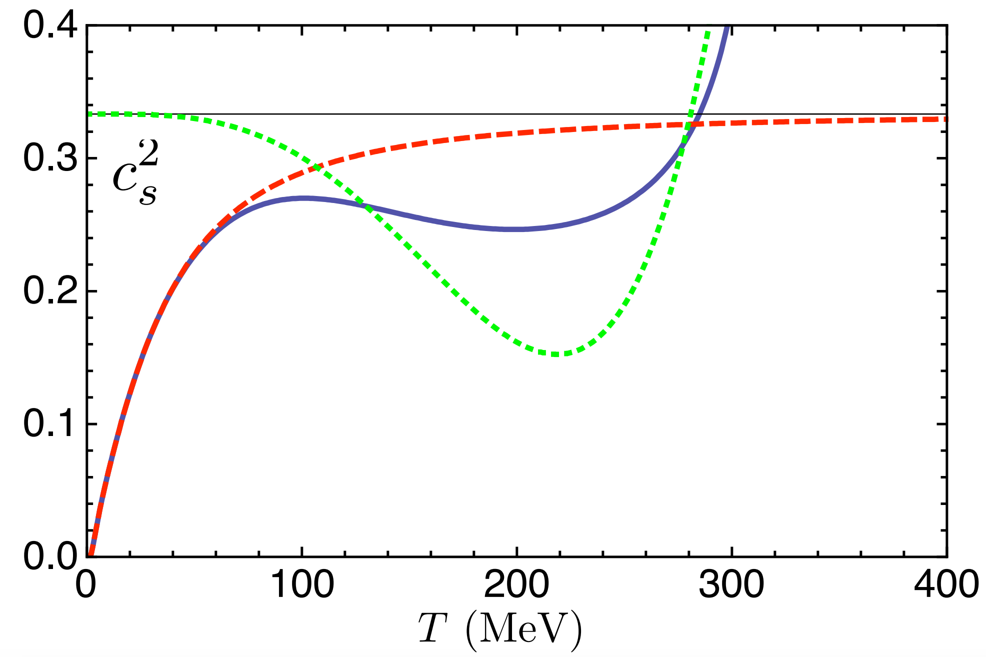
$$\zeta \propto \frac{\Gamma_\sigma}{m_\sigma^2}$$



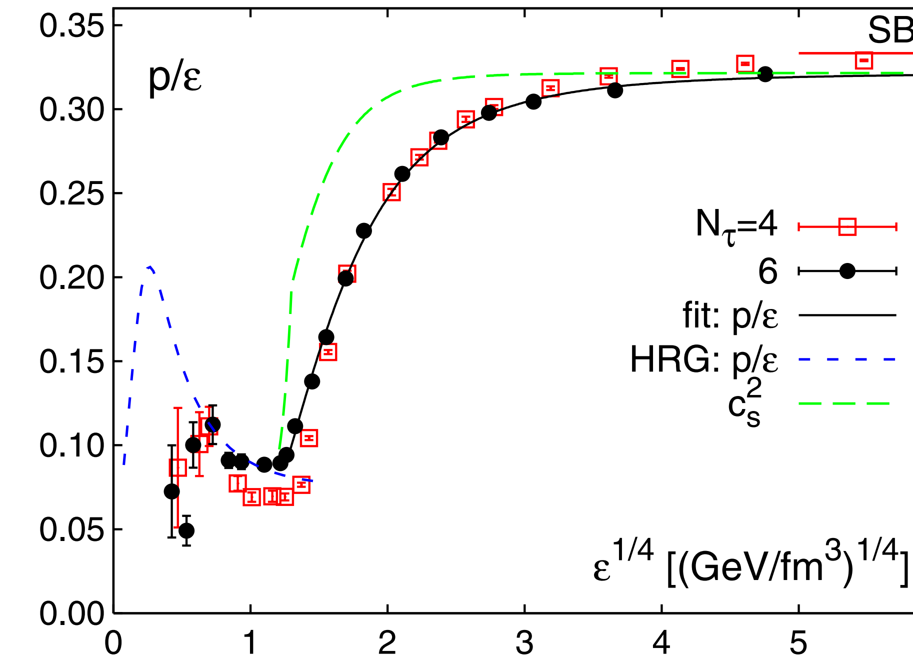
Bulk viscosity of the pion gas (in preparation)

- Heat capacity and speed of sound (ChPT):



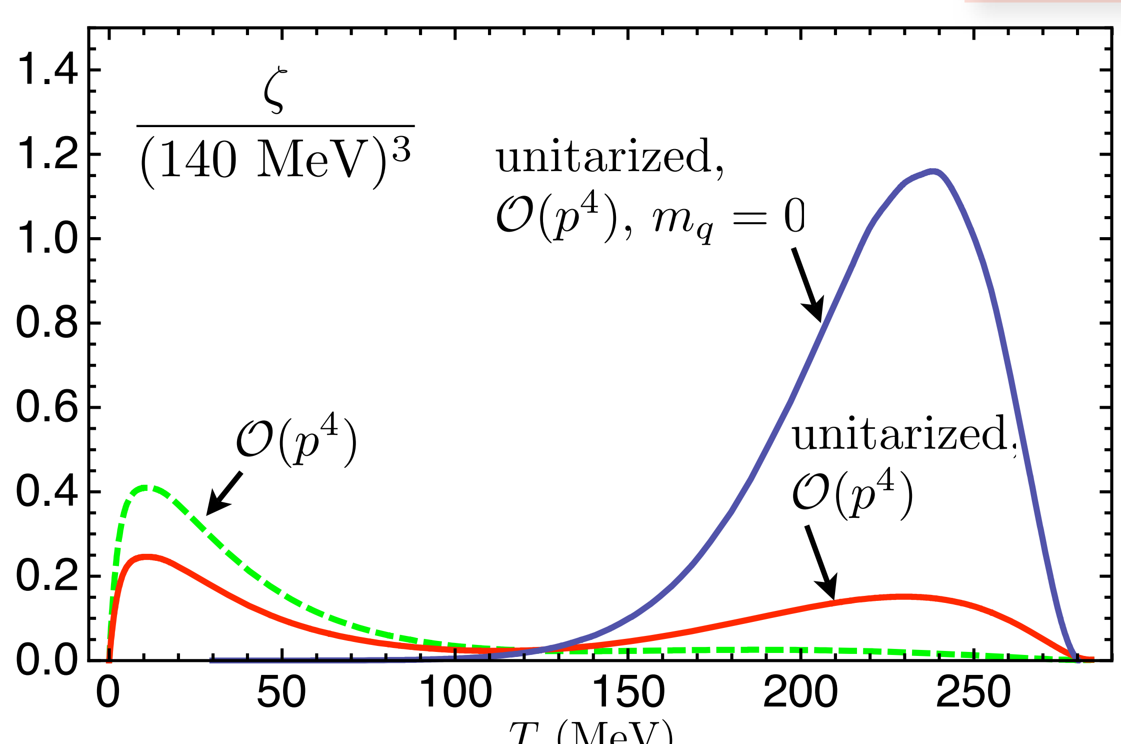


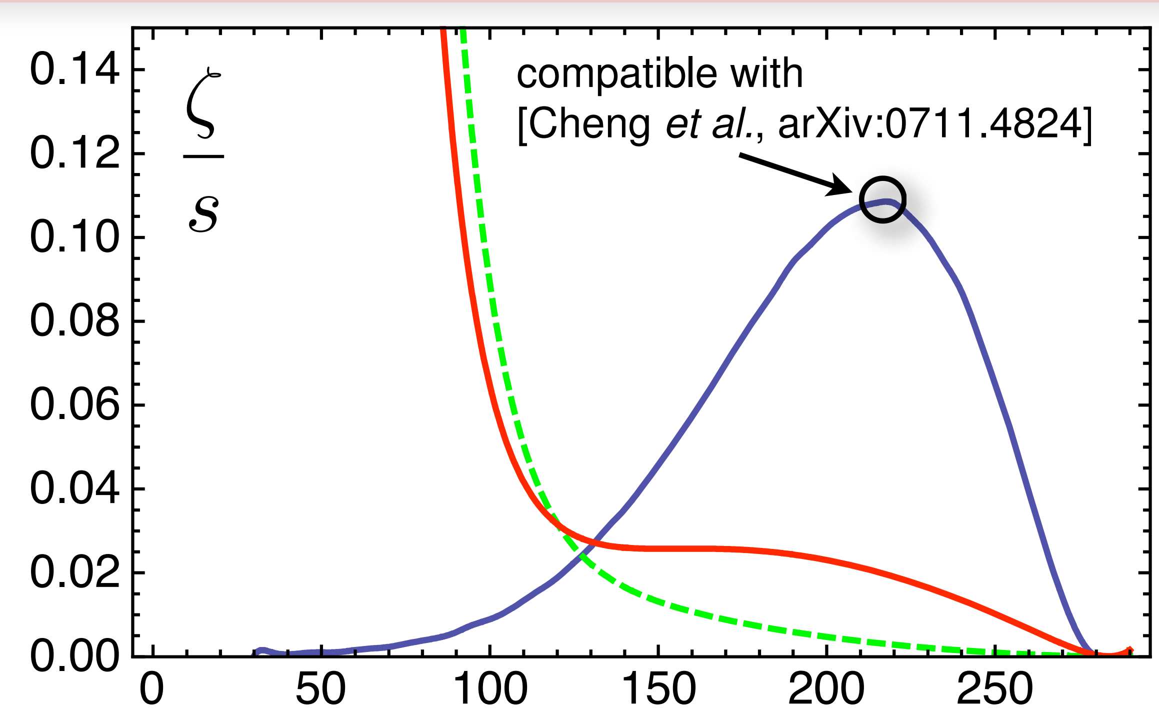
- Lattice (2+1 flavors): [Cheng *et al.*, 08]



- Bulk viscosity (ChPT):

$$\zeta^{(0)} = \int_0^\infty \mathrm{d}p\, \frac{3p^2(p^2/3 - c_s^2 E_p^2)^2}{4\pi^2 T E_p^2 \Gamma_p}\, n_\mathrm{B}(E_p)[1 + n_\mathrm{B}(E_p)]$$





$T \ll M_\pi :\ \zeta^{(0)} \simeq 0.36\, \eta^{(0)}$

$T \simeq M_\pi :\ \zeta^{(0)} \sim 10^{-1} \eta^{(0)}$

$T \gg M_\pi :\ \zeta^{(0)} \sim \left(\frac{1}{3} - v_s^2\right)^2 \eta^{(0)}$

$\Rightarrow \quad \omega_0 \sim 1$ GeV

lattice +sum rule

$\zeta/s|_{T_c} \sim 0.4$

Conclusions

- We cannot apply Weinberg's counting to estimate the contribution of Feynman diagrams to TC at low temperatures.
- Our counting allows us to quickly obtain the leading order contribution for transport coefficients in ChPT at very low temperatures.
- Good agreement with KT analyses, and with phenomenological predictions.
- Resummations may be neccessary for temperatures near T_c. Cancellation of ladders?.

Current lines of work on this topic:

- Extension to SU(3) ($\Rightarrow$ kaons, eta and more resonances). Role of ladder diagrams near T_c.
- More exhaustive study of the conformal properties of the ChPT lagrangian.