Space and Phase-Space saturation:
A simple Bag-Model-inspired picture for a smooth transition to QGP

LF, V. Koch

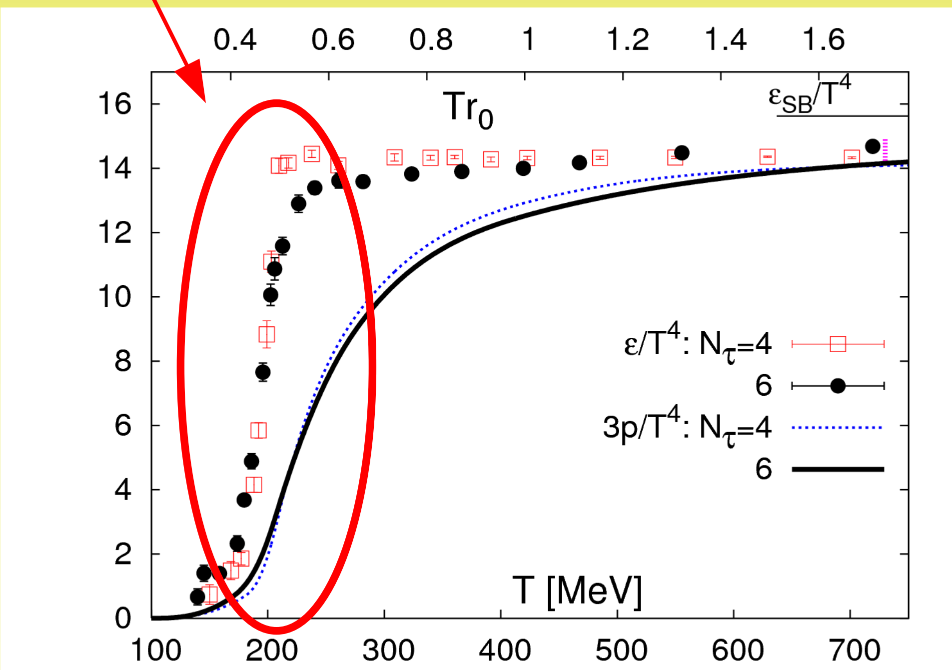
(paper in preparation)

- Introduction.
- Brief reminder on the bag model and compressible bags.
- The partition function of a gas of compressible hadrons.
- Some numerical results
- Conclusions & Outlook.

The crossover transition

Crossover

Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature **443**, 675 (2006)

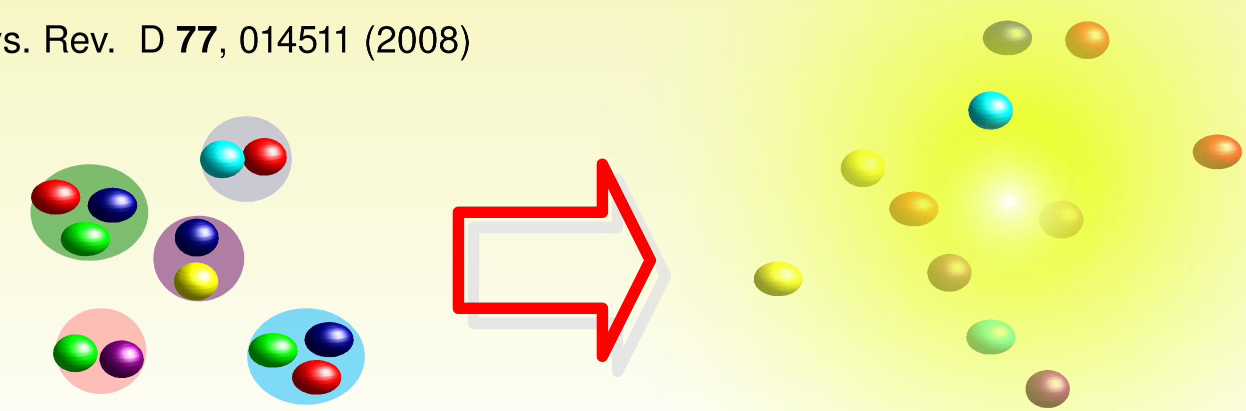


M. Cheng *et al.* Phys. Rev. D **77**, 014511 (2008)

It is now evident that the confined phase of QCD can be described by hadron-gas models.

Around the transition a dual approach should be possible!

Try to understand this transition from the “hadronic side”



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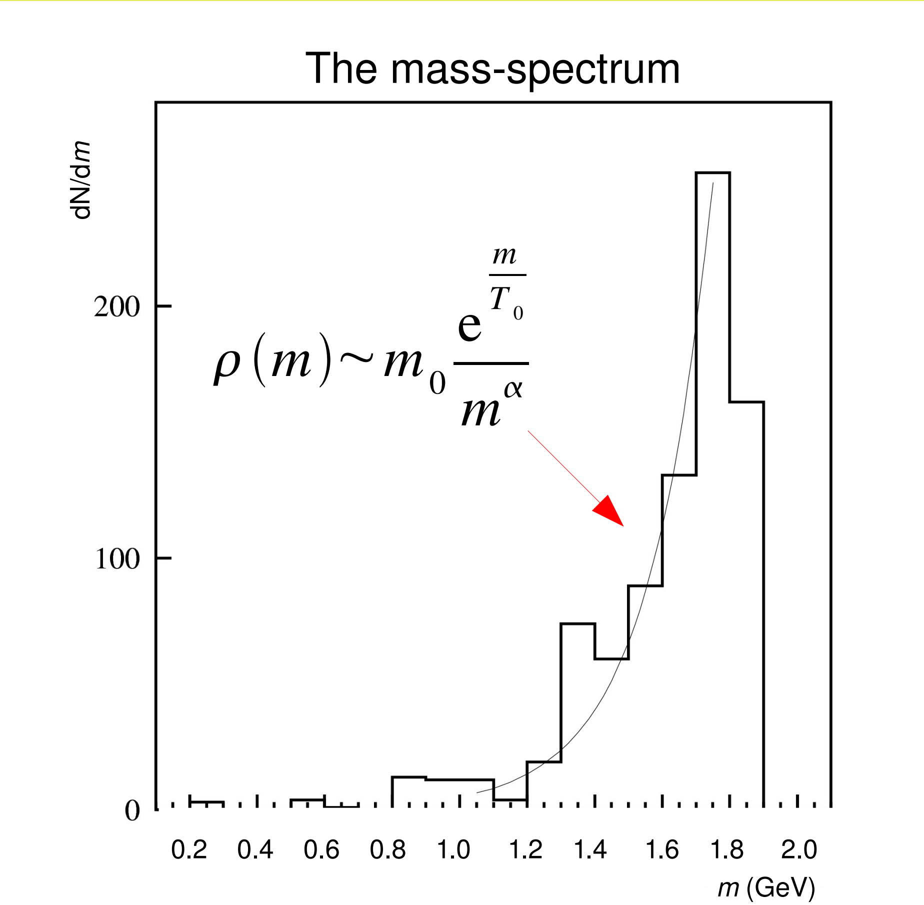
Models of hadrons

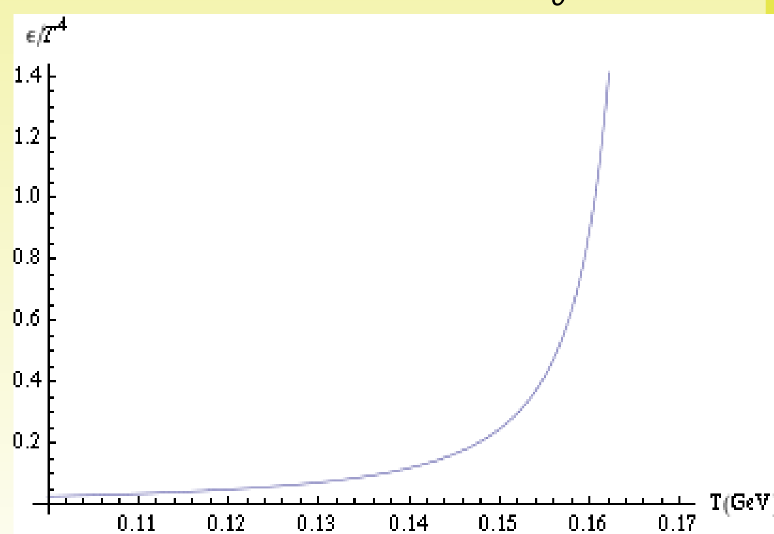


Asymptotically exponential mass-spectrum

$$Z(T,V) = \sum_{N=0}^{\infty} \frac{(V m_0)^N}{N!} \left[\prod_{i=1}^{N} \int_0^{\infty} dm_i \frac{d^3 p_i}{(2\pi)^3} m_i^{-\alpha} \right] \times \exp\left(\sum_{i=1}^{N} \frac{m_i}{T_0} - \sum_{i=1}^{N} \frac{p_i^2/2m_i + m_i}{T} \right)$$

(Boltzmann Statistics N.R. limit)

The partition function diverges at $T=T_0$



- The Statistical Bootstrap Model
  (T_0 is the limiting temperature: the Hagedorn temperature)
- The Bag model
  (T_0 is the transition temperature to the deconfined phase)

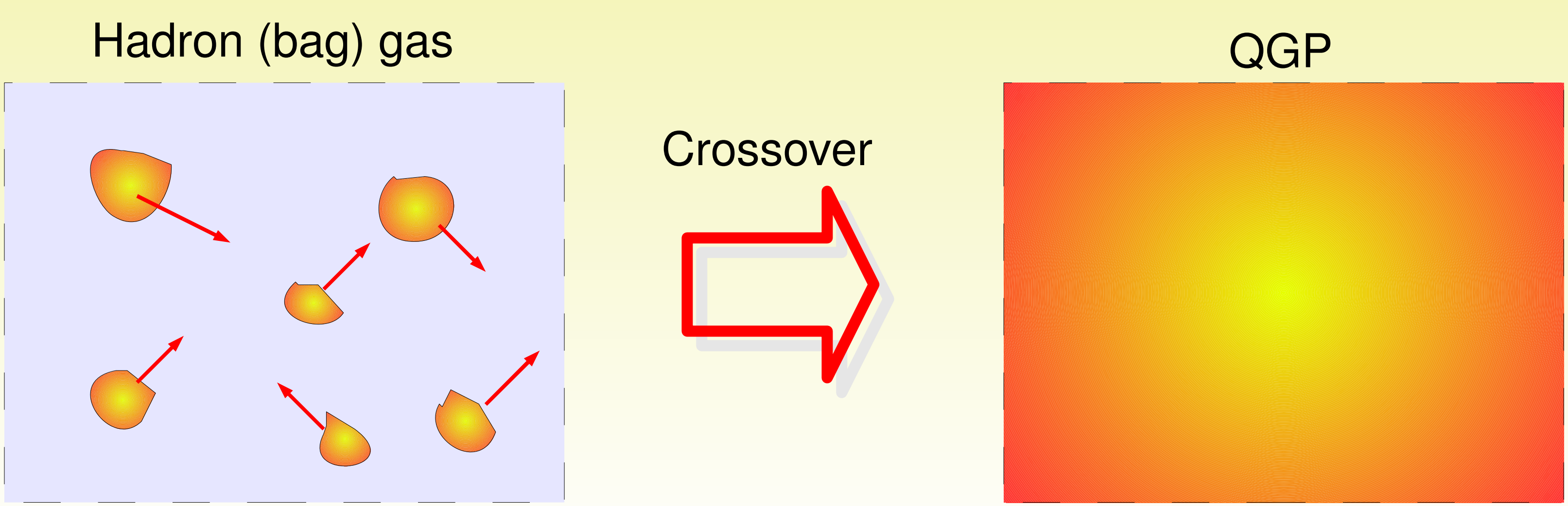
The bag model

In its simplest formulation, hadrons are described as bags of u.r. Ideal gas (the QGP).

The Bag model seems to be a good candidate as it incorporates asymptotic freedom and confinement

<u>It exhibits a first order phase transition</u>

Is there any way to get rid of the excess of d.o.f. without invoking an actual phase transition?



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The bag model

$$m = U + BV_b$$ where $$U = 3p_r V_b$$

(large V_b)

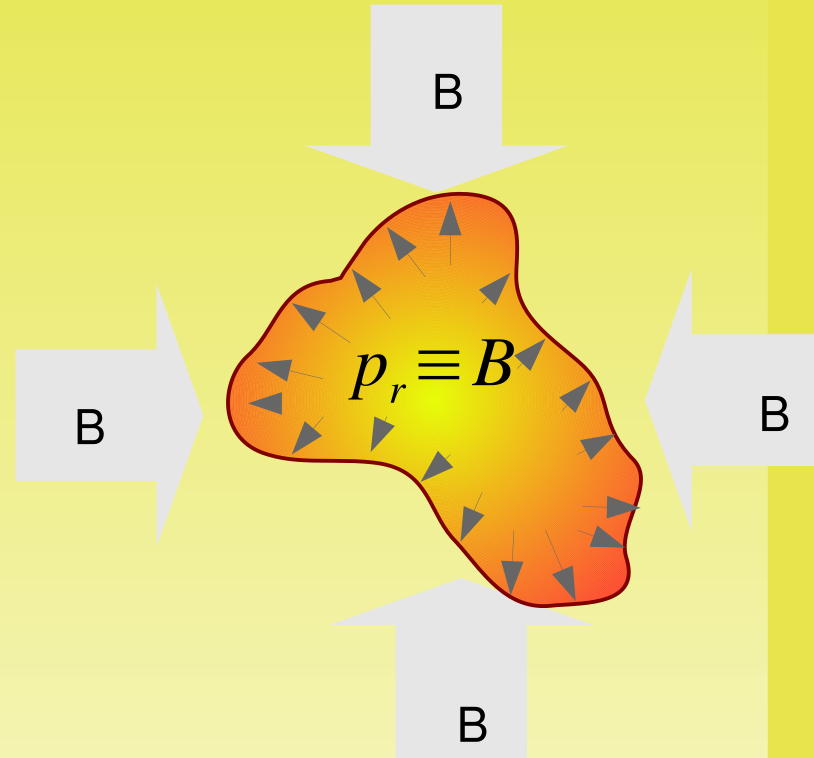
$$m = 4BV_b$$

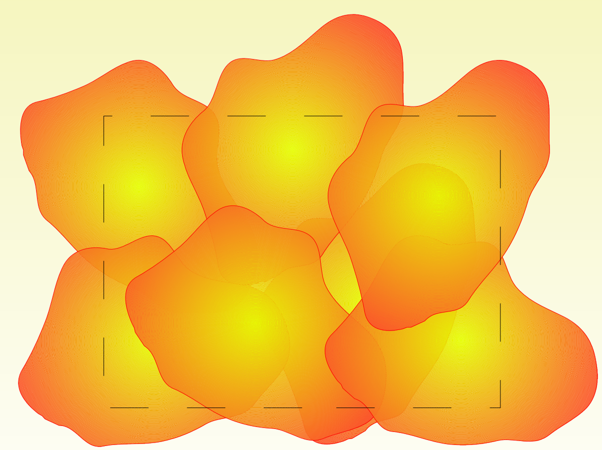
$$S - S_0 = \frac{4U}{3kp_r^{1/4}} = \frac{m}{T_0}$$

(depends on the d.o.f. of the inner field (or u.r. gas))

$$\rho(m) \propto \mathrm{e}^{\frac{m}{T_0}}$$

$$T \to T_0 \quad m \to \infty \quad V_b \to \infty$$





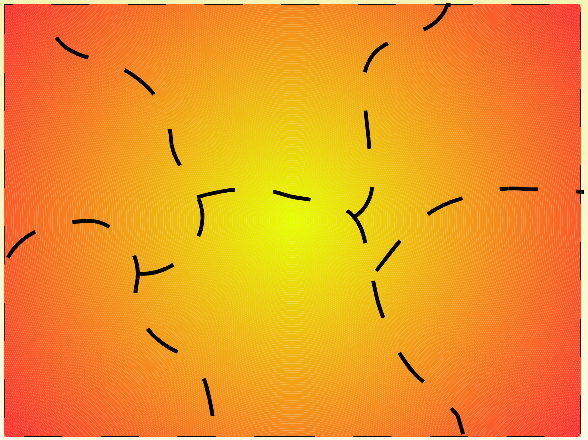
Reduced phase space

$$[\prod_{i=1}^{N}\frac{V}{(2\pi)^3}\int d^3p]\rightarrow[\prod_{i=1}^{N}\frac{1}{(2\pi)^3}\int d^3p](V-\sum_{i=1}^{N}V_i)^N\theta(V-\sum_{i=1}^{N}V_i)$$

M.I. Gorenstein, V.K. Petrov and G. M. Zinovev, Phys. Lett. B **106**, 327 (1981).

Assuming: $V_i=\frac{m_i}{4B}$

$$\epsilon_{max}=4B$$



$Z(V,T)$ well defined at any T

Does not yield the desired thermodynamic at high T

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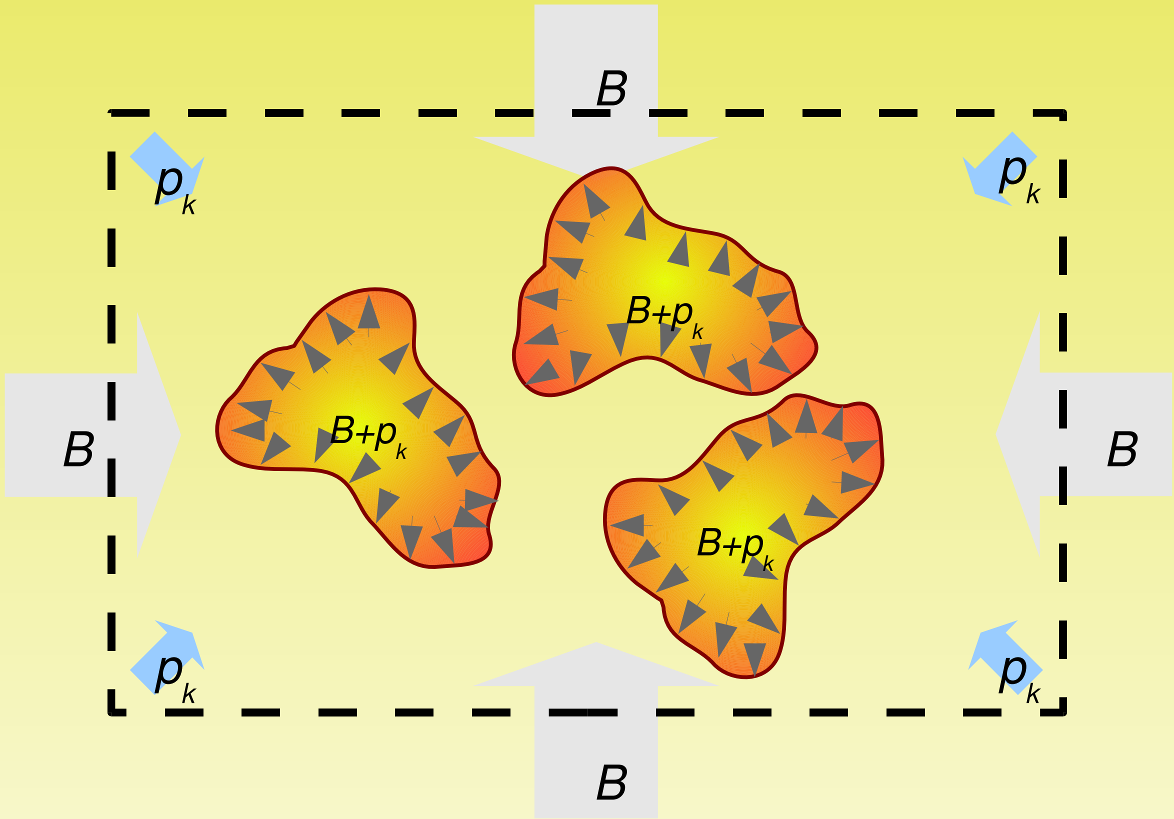
Compressible hadrons

We search for a possible solution by adopting the idea of compressible-bag models



We study the simplest system:

- N.R.
- Boltzmann Statistics
- No Wan Der Waals
- No Surface Energy
- Ideal gas Bag-like hadrons
- Continuous mass-spectrum

Stability Condition:

$$p_r \equiv B \quad \rightarrow \quad p_r(V,T) \equiv B + p_k(V,T)$$

Self Consistency Relation

Associated to the thermal motion of the hadrons themselves

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The isobaric partition function

$$p_r(V,T) \equiv B + p_k(V,T)$$

Must be evaluated from the partition function itself

s=p/T plays the role of an external pressure

$$\boxed{\hat{Z}(T,s) \equiv \int_0^{\infty} dV\, Z(V,T)\, e^{-sV}}$$

$$Z(V,T) \equiv e^{p_k V/T} \Rightarrow \boxed{p_k = sT}$$

In the infinite volume limit the asymptotic behavior of $Z(V,T)$ is defined by the singularity of $\hat{Z}(V,T)$ with the largest real part.

We have two distinct cases depending on the parameter α on

$$\rho(m) = m_0 \frac{e^{\frac{m}{T_0}}}{m^{\alpha}}$$

1. $\alpha \leqslant 5/2$ There is always solution for p_k

2. $\alpha > 5/2$ Phase transition

M.I. Gorenstein, V.K. Petrov and G. M. Zinovev, Phys. Lett. B **106**, 327 (1981).

Here we focus on Case 1

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Numerical analysis of the isobaric partition function

Standard values of the model parameters:

This will be the transition temperature

$$\left(\frac{90}{\pi^2(16+21)}\right)^{1/4} \sim 0.7$$

Chosen on purpose to have the same high-*T* limit of LQCD

$$T_0 \equiv k\,B^{1/4} = 0.17\,GeV$$
$$k = 0.68$$
$$\rightarrow B^{1/4} = 250\,MeV$$

$$m_{cut} = m_\pi = 0.139\,GeV$$
$$\alpha = 3/2$$
$$m_0 = 0.018\,GeV^{\alpha-1}$$

Fixed (for α=3/2) by the actual mass spectrum

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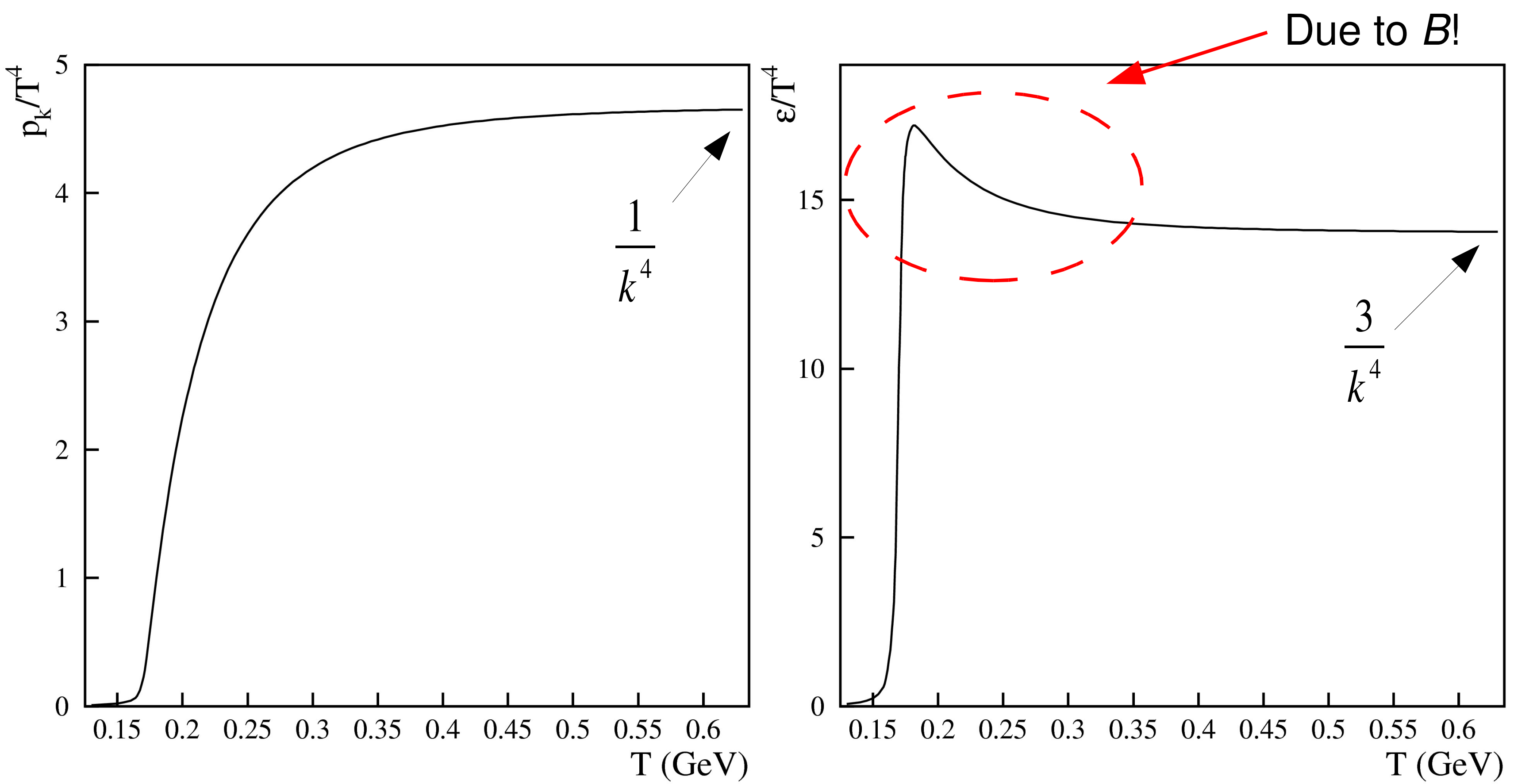
Numerical analysis of the isobaric partition function

$$s_0(T) = f\big(T, s_0(T)\big) = \frac{p_k(\infty, T)}{T}$$

$$\epsilon = T\frac{\partial p_k(\infty, T)}{\partial T} - p_k(\infty, T)$$



Numerical analysis of the isobaric partition function

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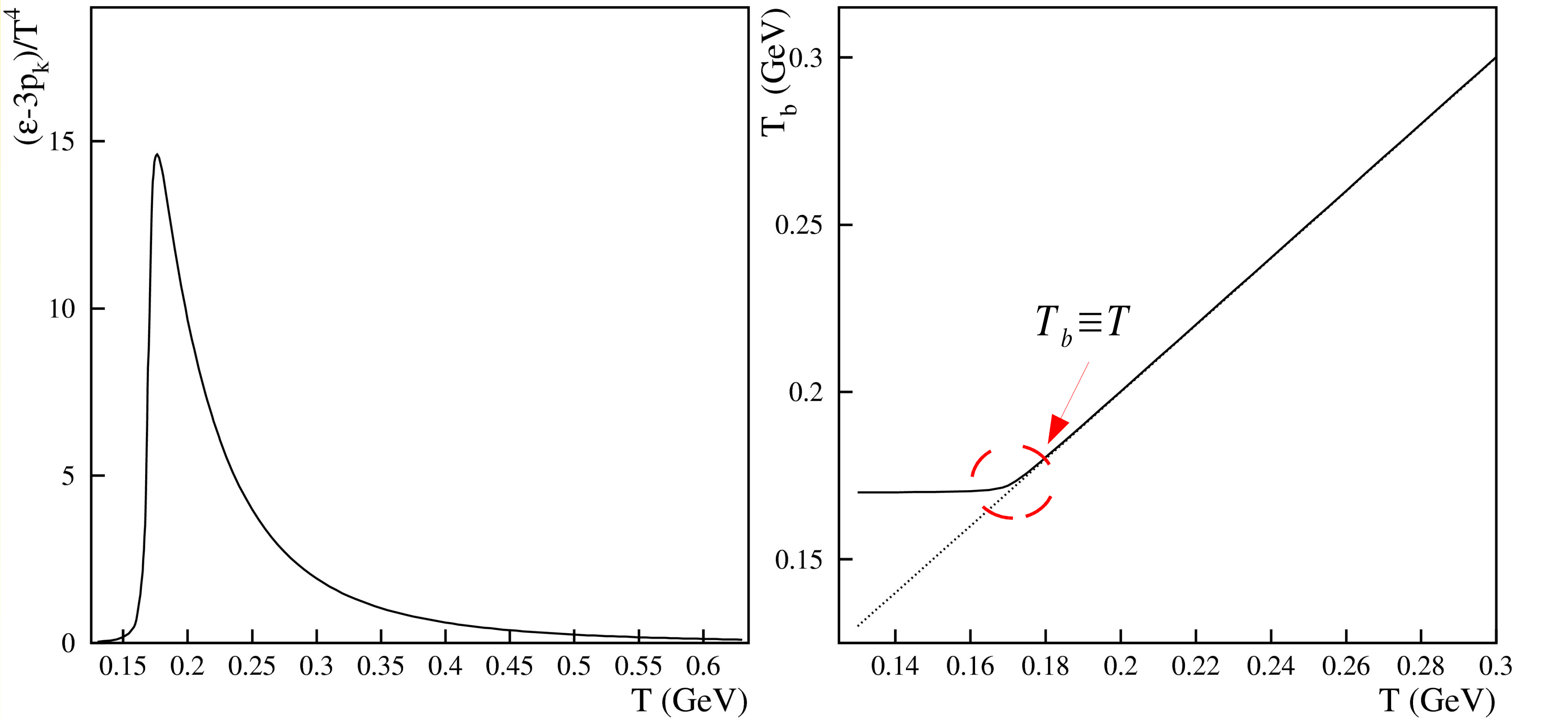
$$\frac{(\epsilon - 3p_k)}{T^4}$$

Effective hadrons inner temperature

$$T_b \equiv k(p_k + B)^{1/4}$$



Numerical analysis of the isobaric partition function

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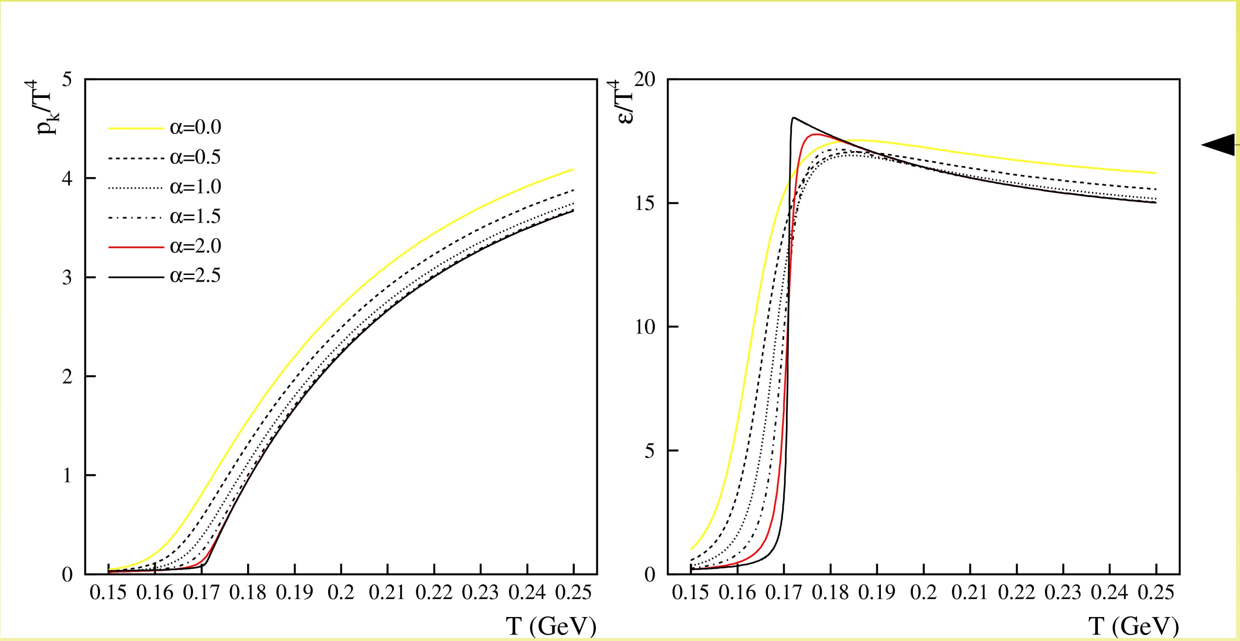
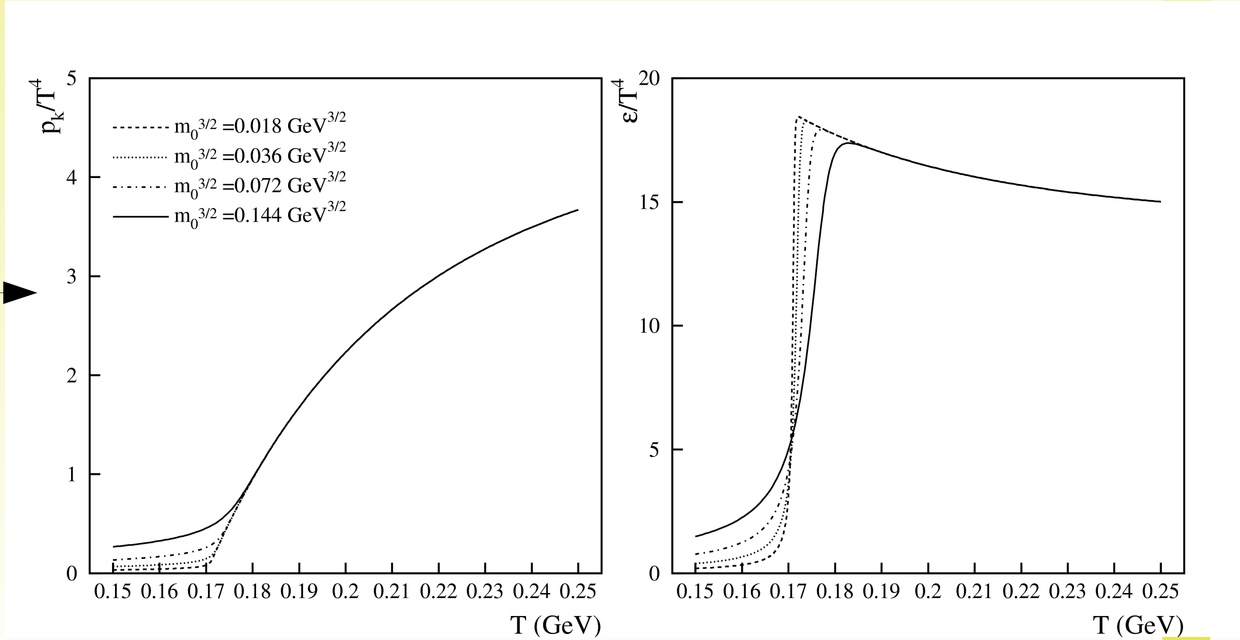
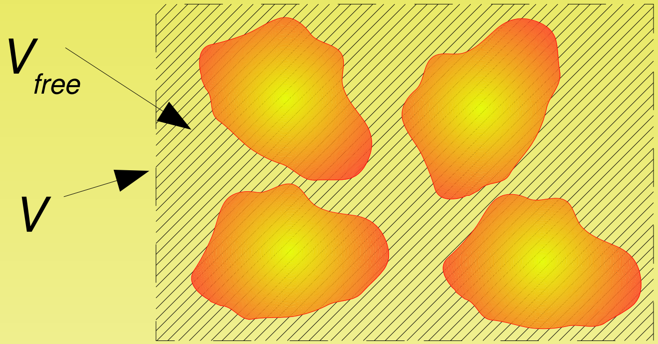


Same as before with various choices of α

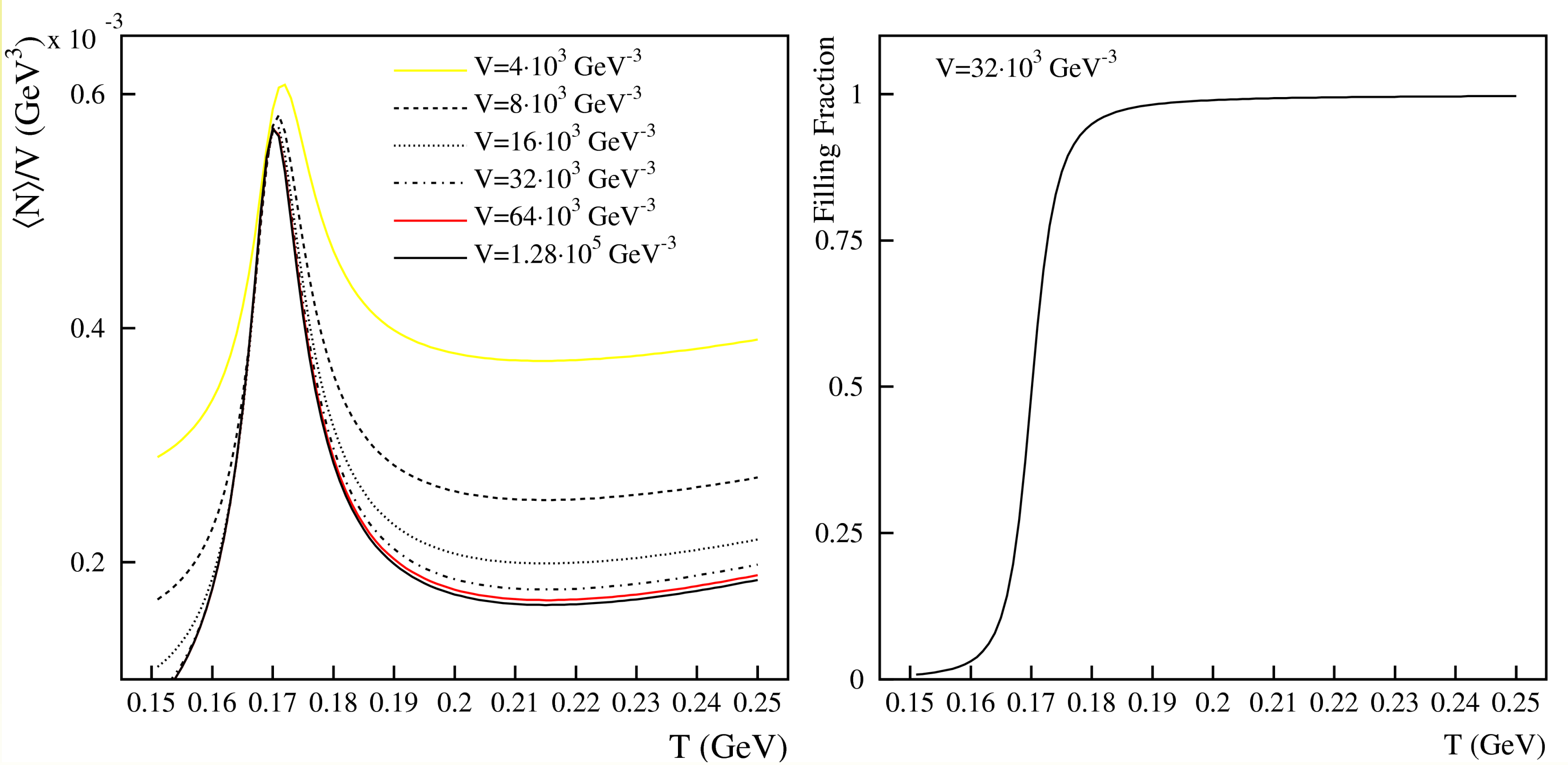
Same as before with various choices of m_0 $(\alpha = 5/2)$



Numerical analysis of the grand canonical partition function

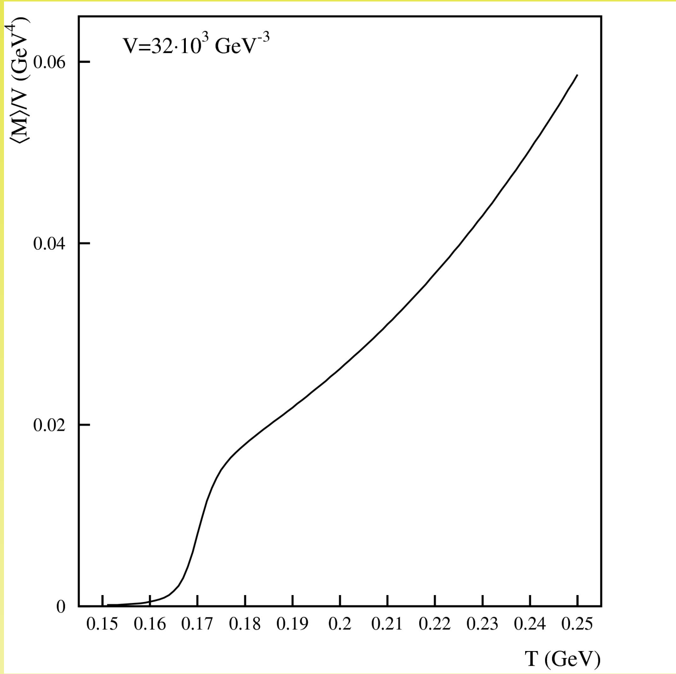


Filling Fraction= 1-V_{free}/V



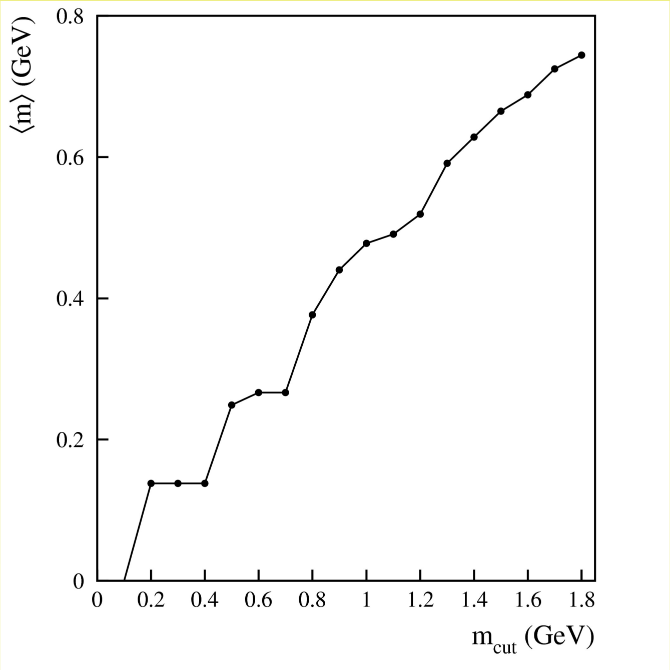
Conclusions & Outlook

- We have adopted the simple idea of hadrons as compressible bags to describe the crossover transition at zero chemical potential.
- We have enforced the stability condition $p_r = B + p_k$
- The results of the model are in a fair qualitative agreement with the Lattice QCD data.
- There are still issues to understand and to discuss.
- In the future, a more realistic approach would be desirable.
- Generalization to finite Baryochemical potential.

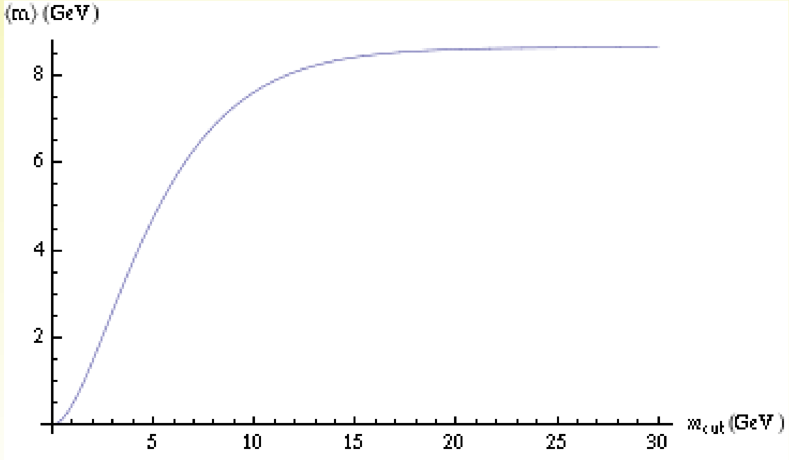


$$T = 160\,MeV \rightarrow \langle m \rangle = 2.8\,GeV$$
$$T = 170\,MeV \rightarrow \langle m \rangle = 14\,GeV$$

Actual hadron gas at 170 MeV: average mass as a function of the upper cut-off



Hgedorn spectrum at 160 MeV: average mass as a function of the upper cut-off



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The partition function

$$V_i \rightarrow \frac{U_i}{3 p_r}$$

$$m_i \rightarrow U_i\left(1+\frac{B}{3 p_r}\right)$$

$$S_i = \frac{m_i}{T_0} \rightarrow \frac{4 U_i}{3 k p_r^{1/4}}$$

After integrating over particle's momenta...

$$Z(T,V) = \sum_{N=0}^{\infty} \left(\frac{T}{2\pi}\right)^{3N/2} \frac{m_0^N}{N!} \left[\prod_{i=1}^{N} \int_0^{\infty} dU_i \left(U_i + B\frac{U_i}{3 p_r}\right)^{3/2-\alpha}\right]$$
$$\times \exp\left\{\left[\frac{4}{3 k p_r^{1/4}} - \frac{1}{T}\left(1+\frac{B}{3 p_r}\right)\right]\sum_{i=1}^{N} U_i\right\} \left(V - \sum_{i=1}^{N}\frac{U_i}{3 p_r}\right)^N \theta\left(V - \sum_{i=1}^{N}\frac{U_i}{3 p_r}\right)$$

$$U_i \rightarrow m_i = \frac{4}{3} U_i$$

Lower bound $m_{cut} = m_{\pi}$

$$Z(T,V) = \sum_{N=0}^{\infty} \left(\frac{T}{2\pi}\right)^{3N/2} \frac{m_0^N}{N!} \left[\prod_{i=1}^{N} \int_{m_{cut}}^{\infty} dm_i \left(\frac{3}{4} m_i + B\frac{m_i}{4 p_r}\right)^{3/2-\alpha}\right]$$
$$\times \exp\left\{\left[\frac{1}{k p_r^{1/4}} - \frac{1}{T}\left(\frac{3}{4}+\frac{B}{4 p_r}\right)\right]\sum_{i=1}^{N} m_i\right\} \left(V - \sum_{i=1}^{N}\frac{m_i}{4 p_r}\right)^N \theta\left(V - \sum_{i=1}^{N}\frac{m_i}{4 p_r}\right)$$

Note: for $p_r = B$ one recover the previous partition function with the new phase space

The isobaric partition function

$$\hat{Z}(T,s) \equiv \int_0^\infty dV\, Z(V,T)\, e^{-sV}$$

Must be evaluated from the partition function itself

s=p/T plays the role of an external pressure

$$\hat{Z}(T,s) \equiv \int_0^\infty dV\, Z(V,T)\, e^{-sV}$$

$$Z(V,T) \equiv e^{p_k V/T}$$

$$\boxed{p_k = sT}$$

The integral in *dV* can be solved and gives:

$$\hat{Z}(T,s) = \frac{1}{s} \sum_{N=0}^{\infty} \left[\frac{f(T,s)}{s} \right]^N = \frac{1}{s - f(T,s)}$$

$$f(T,s) = m_0 \left(\frac{T}{2\pi} \right)^{3/2} \int_{m_{cut}}^{\infty} dm \left(\frac{3}{4} m + B \frac{m}{4(B+sT)} \right)^{3/2-\alpha} \exp\left\{ \frac{m}{k(B+sT)^{1/4}} - \frac{m}{T} \right\}$$

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The infinite volume limit

In the infinite volume limit the asymptotic behavior of $Z(V,T)$ is defined by the singularity of $\hat{Z}(V,T)$ with the largest real part.

We have two distinct cases depending on the parameter α on

$$\rho(m) = m_0 \frac{e^{\frac{m}{T_0}}}{m^{\alpha}}$$

1. $\alpha \leqslant 5/2$
2. $\alpha > 5/2$

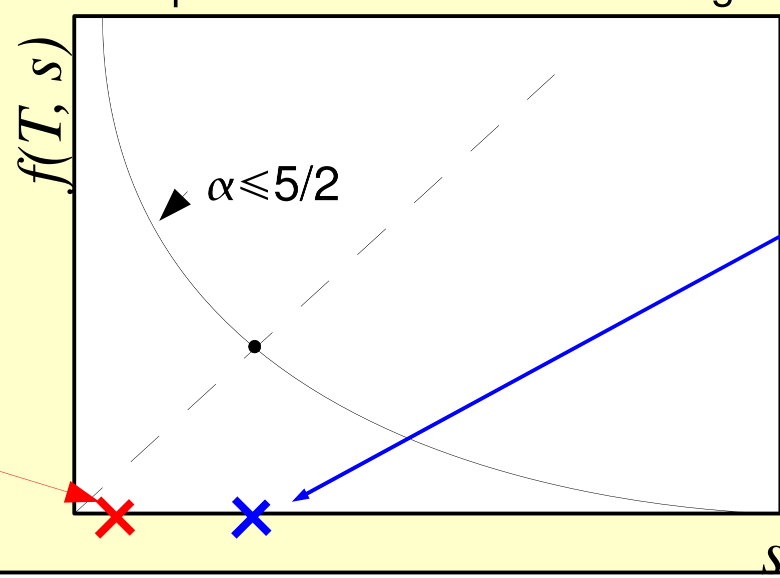
Case 1

$$\hat{Z}(T,s) = \frac{1}{s}\sum_{N=0}^{\infty}\left[\frac{f(T,s)}{s}\right]^N = \frac{1}{s - f(T,s)}$$

The isobaric partition function has two singularities:

1) The divergence of the function *f* itself when the exponent of the integrand vanishes:

$$s_f(T) = \frac{T^3}{k^4} - \frac{B}{T}$$



2) The pole:

$$s_0(T) = f\left(T, s_0(T)\right)$$

Is the solution for the pressure $p_k(\infty,T)$

This one is alwais the rightmost!

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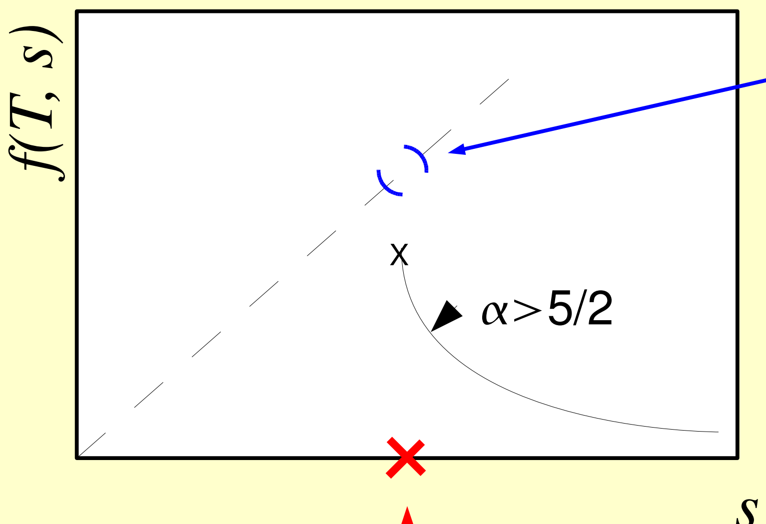
The infinite volume limit

Case 2 $\left(\alpha > 5/2\right)$ $\qquad \hat{Z}(T,s) = \frac{1}{s}\sum_{N=0}^{\infty}\left[\frac{f(T,s)}{s}\right]^N = \frac{1}{s - f(T,s)}$

$$f(T,s) = m_0\left(\frac{T}{2\pi}\right)^{3/2}\int_{m_{cut}}^{\infty} dm\left(\frac{3}{4}m + B\frac{m}{4(B+sT)}\right)^{3/2-\alpha}\exp\left\{\frac{m}{k(B+sT)^{1/4}} - \frac{m}{T}\right\}$$



For sufficiently large T, the pole:

$$s_0(T) = f\left(T, s_0(T)\right)$$

does not exist and there is **no solution for the system's pressure!**

$$s_f(T) = \frac{T^3}{k^4} - \frac{B}{T}$$

The function f is finite in $s{=}s_f$,

while diverges for $s{<}s_f$.

This situation has been discussed in:

M.I. Gorenstein, V.K. Petrov and G. M. Zinovev, Phys. Lett. B **106**, 327 (1981).

possible mechanism for a **phase transition**.

Here we focus on Case 1

The grand canonical partition function

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$$Z(T,V)=\sum_{N=0}^{\infty} Z_N(T,V)$$

Now the pressure of N hadrons

$$Z_N(T,V)=\left(\frac{T}{2\pi}\right)^{3N/2}\frac{m_0^N}{N!}\left[\prod_{i=1}^{N}\int_{m_{cut}}^{\infty} dm_i\left(\frac{3}{4}m_i+B\frac{m_i}{4p_{r,N}}\right)^{3/2-\alpha}\right]$$

$$\times\exp\left\{\left[\frac{1}{k\,p_{r,N}^{1/4}}-\frac{1}{T}\left(\frac{3}{4}+\frac{B}{4p_{r,N}}\right)\right]\sum_{i=1}^{N}m_i\right\}\left(V-\sum_{i=1}^{N}\frac{m_i}{4p_{r,N}}\right)^{N}\theta\left(V-\sum_{i=1}^{N}\frac{m_i}{4p_{r,N}}\right)$$

We **maximize the integrand with respect** $\boldsymbol{p_{r,N}}$

$$\frac{T}{k\langle p_{r,N}\rangle^{1/4}}\langle p_{r,N}\rangle=B+\frac{NT}{V-\sum_{i=1}^{N}\frac{m_i}{4\langle p_{r,N}\rangle}}$$

$\langle T_b\rangle$

$V-\sum_{i=1}^{N}V_i$

For $T/\langle T_b\rangle=1$ This **automatically gives** the **stability condition**:

$$\langle p_{r,N}\rangle\equiv B+\langle p_{k,N}\rangle$$

$$\langle p_{k,N}\rangle\equiv\frac{NT}{V-\sum_{i=1}^{N}\frac{m_i}{4\langle p_{r,N}\rangle}}=\frac{NT}{V-\sum_{i=1}^{N}V_i}$$

The **self-consistency relation.**