# Perturbative Method at High Dense Quark Matter

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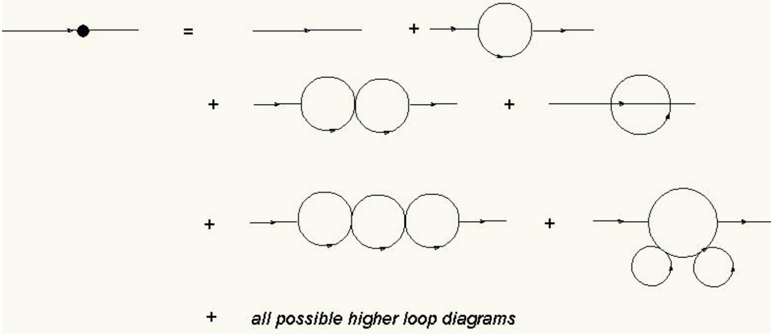
# Outline

- *Method*
- *Former Way*
- *Improved Way*
- *Another Interesting Way*

## Why Use The Perturbative Method

- is a classical and elegant method
- has been developed in many area, such as large $N_c$
- has a natural small number, coupling constant $g$ at high density quark matter
- lattice fails at high density quark matter

# The Meaning Of The HTL




the usual connection of the naive perturbative analysis of QCD between the order of the loop expansion and powers of $g$ is lost at high temperature. Since the effects of leading order in $g$ arise from every order in the loop expansion the resummation is necessary to use to get the effective propagators

# Hard Thermal Loop

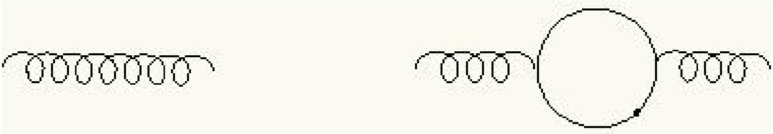


**Figure:** the left one $\sim \frac{1}{p^2}$; the right one with hard loop $\sim \frac{1}{p^2}\frac{g^2T^2}{p^2}$

When any external leg is hard $\sim T$, these diagrams are at least g times the tree amplitude, and are part of the usual perturbative corrections. When every external momentum is soft $\sim gT$, however, $\frac{g^2T2}{p^2}$ is of order **1**, thus hard thermal loops are as important as the tree diagram.

## The Dictionary Between HTL And HDL

### Temperature

- hard momentum $\sim T$
- soft momentum $\sim gT$
- integration $\int k n_{BE}(k, T)\mathrm{d}k = 2\int k\widetilde{n}_{FD}(k, T)\mathrm{d}k = \frac{\pi^2 T^2}{6}$
- thermal mass $\sim gT$

### Density

- hard momentum $\sim \mu$
- soft momentum $\sim g\mu$
- integration $\int k\widetilde{n}_{FD}(k, \mu)\mathrm{d}k = \frac{\mu^2}{2}$
- thermal mass $\sim g\mu$

# The Difference Between HTL And HDL

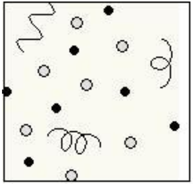


**Figure:** QCD matter at high temperature

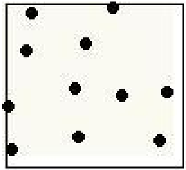
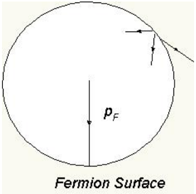


**Figure:** QCD matter at high density




For the Pauli blocking, at high density, both the fermion momentums of the external leg and the internal loop are hard while the momentum of the external leg is soft at high temperature.

## FSQ: An Approvement From HDL

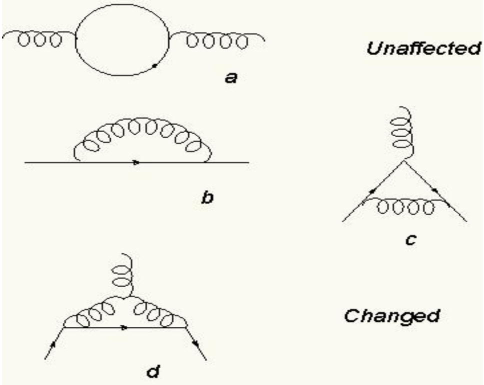



**Figure:** leading one-loop feynman diagrams in QCD

Fermion Surface Quark (FSQ) is a more physical considered method at high density quark matter. Now, fermion self-energy, Abelian and non-Abelian vertex corrections are changed comparing with HDL. However, the dressed gluon propagator was unaffected at this level.

## The Results

| | *FSQ* | *HDL* |
|---|---|---|
| $m^{thermal}$ | $\frac{g^2\mu^2}{8\pi^2}$ | $\frac{g^2\mu^2}{8\pi^2}$ |
| $\Sigma_+^{fermion}$ | $\frac{-ig^2\mu}{12\pi^2 p}\left[\mu + \frac{\omega(\omega-p)}{2p}\log\left(\frac{\omega+p}{\omega-p}\right)\right]$ | $\frac{-ig^2\mu}{12\pi^2 p}\left[\mu - \frac{\mu(\omega-p)}{2p}\log\left(\frac{\omega+p}{\omega-p}\right)\right]$ |
| $\Gamma_+^{Abelian}$ | $\frac{-ig^3\mu^2}{16\pi^2 p^2}\left\{3 - \frac{\omega}{p}\log\frac{\omega+p}{\omega-p}\right\}$ | $\frac{-ig^3\mu^2}{16\pi^2 p^2}\left\{2 - \frac{\omega}{p}\log\frac{\omega+p}{\omega-p}\right\}$ |
| $\Gamma_+^{nonAbe}$ | $\frac{-ig^3\mu^2}{16\pi^2 p^2}\left\{1 - \frac{\omega}{p}\log\frac{\omega+p}{\omega-p}\right\}$ | $\frac{-ig^3\mu^2}{16\pi^2 p^2}\left\{2 - \frac{\omega}{p}\log\frac{\omega+p}{\omega-p}\right\}$ |

## Another Interesting Small Number In Soft Loop

- bare fermion propagator is still useful. Note: $p_4 \sim p_{\parallel} \sim \frac{p_\perp^2}{2\mu}$

$$S^E(p_4, p_{\parallel}, p_\perp) = \frac{-\mathrm{i}}{p_4 + p_{\parallel} + \frac{p_\perp^2}{2\mu}} \tag{1}$$

- dressed gluon propagator
  - longitudinal part in Euclid space which is deeply screened

$$D^E(k_4, k_{\parallel}, k_\perp) = \frac{-\mathrm{i}}{k_4^2 + k_{\parallel}^2 + k_\perp^2 + 2m^2} \tag{2}$$

  - transverse part in Euclid space which is poorly screened via Landau damping. Note: $k_4 \ll k_\perp \ll m$ and $p_\perp \sim k_\perp$

$$D^E(k_4, k_{\parallel}, k_\perp) = \frac{-\mathrm{i}}{k_{\parallel}^2 + k_\perp^2 + \frac{\pi}{2} m^2 \frac{|k_4|}{k_\perp}} \approx \frac{-\mathrm{i}}{k_\perp^2 + \frac{\pi}{2} m^2 \frac{|k_4|}{k_\perp}} \tag{3}$$

when $|\overrightarrow{k}| \sim (m^2 k_4)^{1/3} \gg k_4 (= \omega)$, the propagator became large. Moreover, the small number is not $g$, but $\epsilon = \omega/m$

# The Future Work With These Results

- put the new results in the Color-Superconductivity which is a developing area in particles and nuclear physics
- calculate the gauge dependent terms
- analysis some particularly simple Ward identities

# Thank You

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