# Viscous hydrodynamics for dissipative relativistic fluids

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Presented at the workshop on

Hot quarks 2008, Denver, Co, Aug. 18-23,2008

Supported by DOE

**References:**

H. Song and U. Heinz, Phys.Rev.C78:024902 (2008). arXiv:0805.1756 [nucl-th].

H. Song and U. Heinz, Phys.Rev.C77:064901 (2008).

H. Song and U. Heinz, Phys. Lett. B658, 279 (2008).

08/18/2008

# The QGP shear viscosity

- ideal hydro is a great success in describing RHIC data: spectra and v2

- quantum mechanics excludes the possibilities of a perfectly ideal fluid with zero viscosity-to-entropy ratio

- Weakly coupled QCD prediction: $T >> \Lambda_{QCD}$ P. Arnold, G.Moore & L.Yaffe '00,'03

$$\eta = \frac{T^3}{(\alpha_s)^2 \ln(1/\alpha_s)}$$

However, to show liquid behavior the QGP must be a strongly coupled system.

- Strongly coupled AdS/CFT prediction:

AdS/CFT correspondence: gauge/gravity duality

4d gauge theory at strong coupling ⟷ 5d gravity at weak coupling

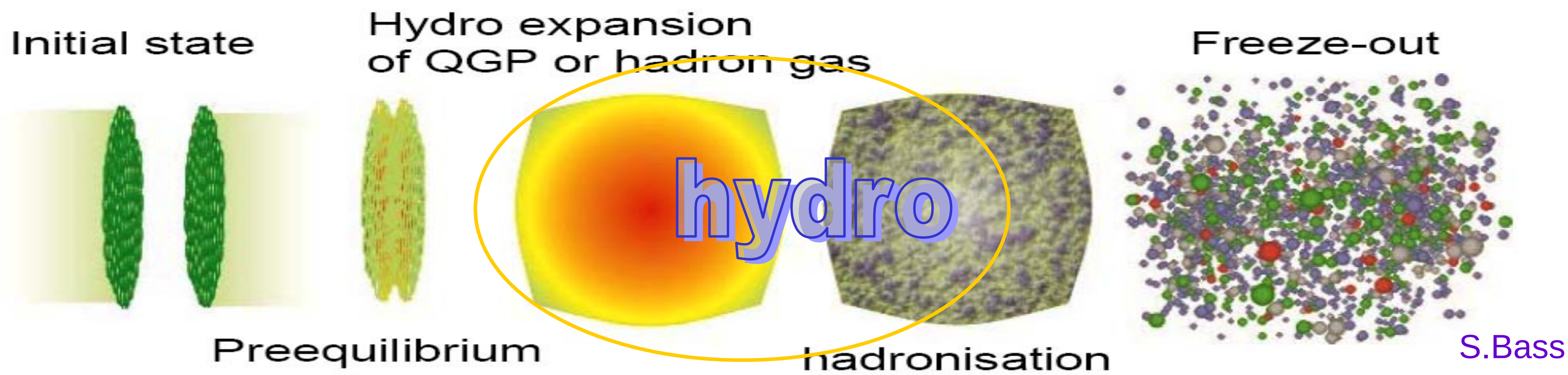
N=4 SYM ⟷ Type IIB superstring theory on $AdS_5 \times S^5$

$$\eta / s \geq 1/4\pi \approx 0.08$$

D.T. Son et al. '01,'05
(not related to real QCD)

**To extract the QGP viscosity from experimental data, we need viscous hydrodynamics**

# Viscous hydrodynamics




Conservation laws:

$\partial_\mu T^{\mu\nu}(x) = 0$ $\qquad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu} + \pi^{\mu\nu}$

Evolution equations for shear pressure tensor $\pi^{\mu\nu}$ :

$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$ -simplified Israel-Stewart eqn.

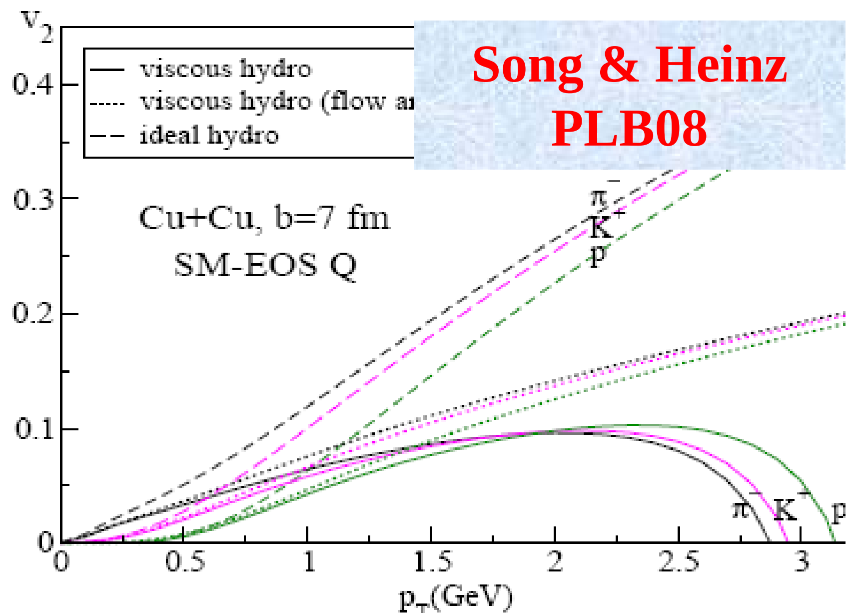
Input: "EOS" $\varepsilon = \varepsilon(p, n)$ initial conditions and final conditions

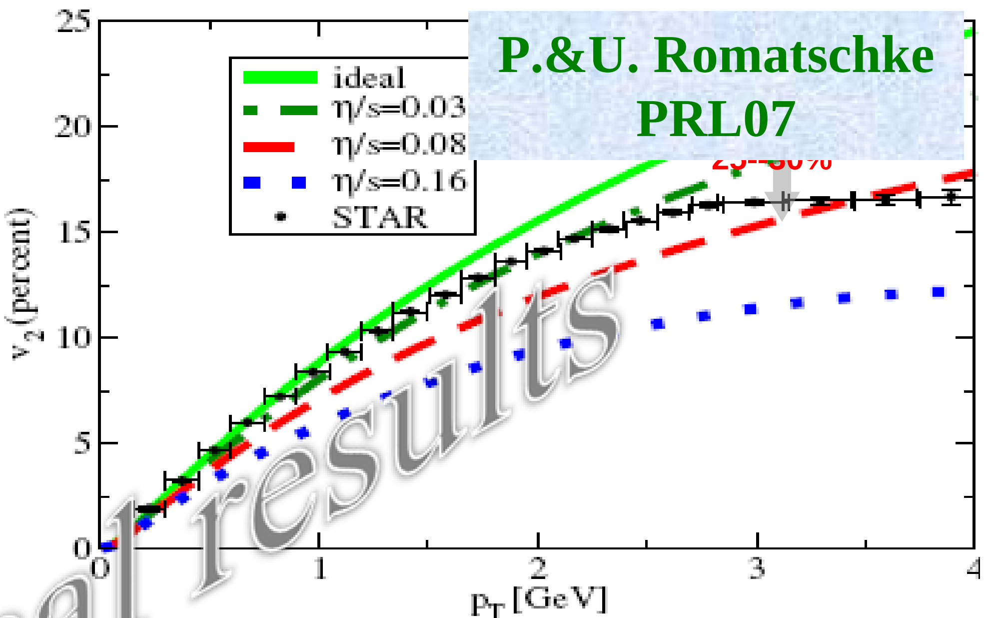
With $\eta \to 0$ **viscous hydrodynamics** reduces to **ideal hydrodynamics**

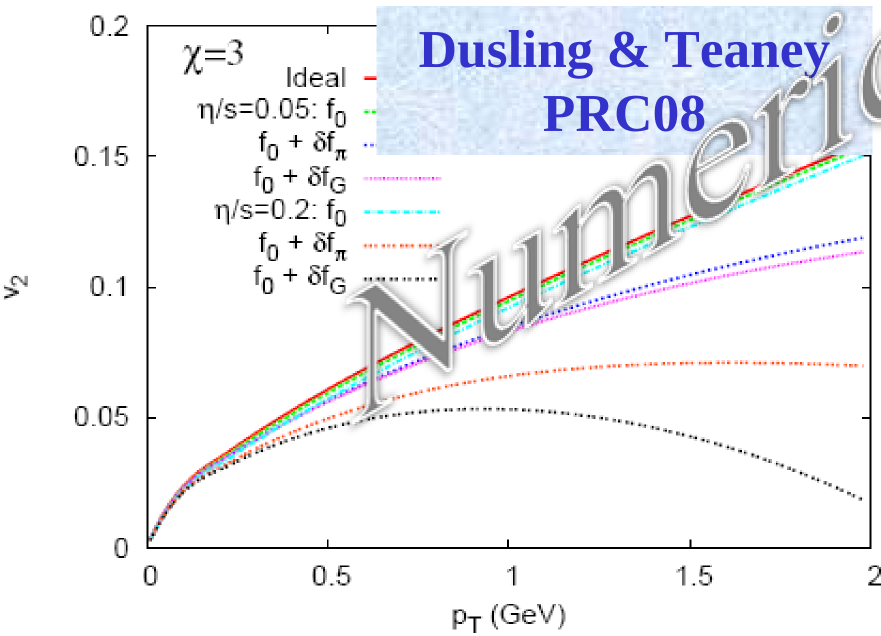
A further simplification: Bjorken approximation $v_z = z/t$

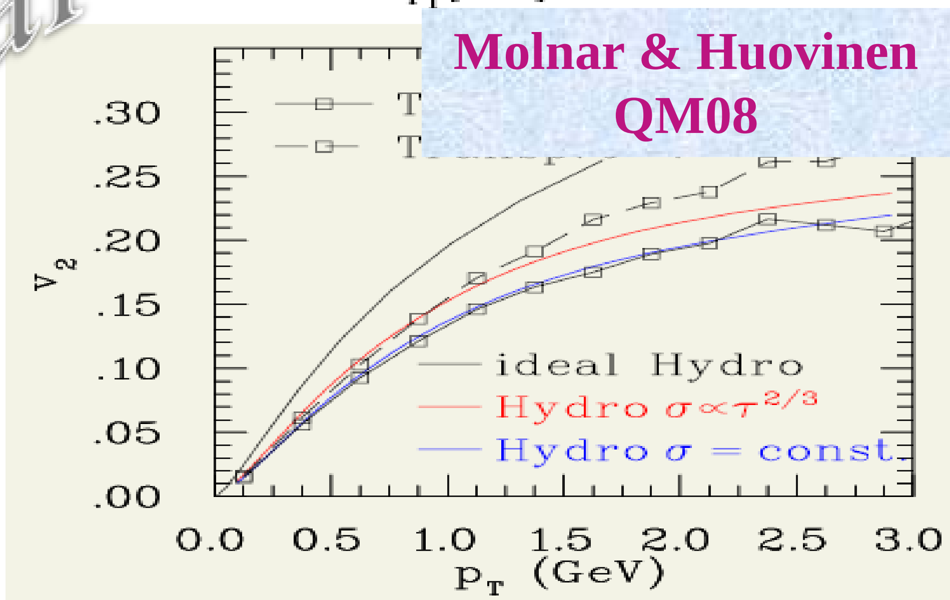
Reduces (3+1)-d hydrodynamics to (2+1)-d hydrodynamics $(\tau, x, y, \eta)$

# (2+1)-d viscous hydrodynamics




Numerical results

# (2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

PRL'07 **Au+Au,** $T_{dec}$ = **150MeV** (EOS L* here is the quasi-particle one based on lattice QCD)

-Song & Heinz: simplified I-S eqn. & full I-S eqn. EOS I SM-EOS Q EOS L

PLB'08 & arXiv:0712.3715[nucl-th] **Cu+Cu,** simplified I-S eqn., $T_{dec}$ = **130MeV**

(**Au+Au**, **Cu+Cu**, system size effects, full I-S eqn. vs. simplified I-S eqn., EOS L etc, in preparation)

-Dusling & Teaney: Őttinger-Grmela (O-G) eqn. EOS I

PRC'08 **Au+Au,** decoupling by scattering rate, arXiv:0803.1262 [nucl-th], (dilepton production)

-Huovinen & Molnar: full I-S eqn. EOS I

QM08 talk: comparing the results from viscous hydro and from transport model

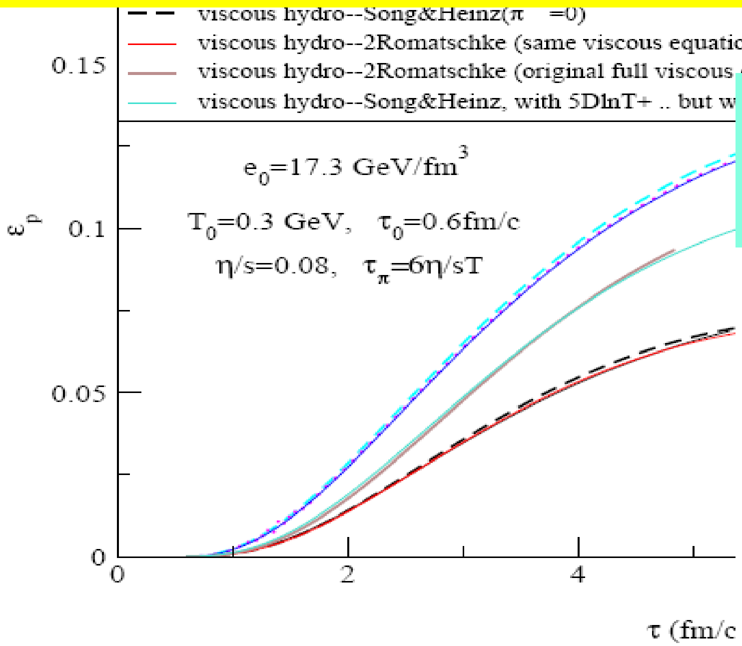
-Chaudhuri: simplified I-S eqn. EOS I EOS Q

arXiv:0708.1252 [nucl-th], arXiv:0801.3180 [nucl-th], arXiv:0803.0643 [nucl-th] **Au+Au**

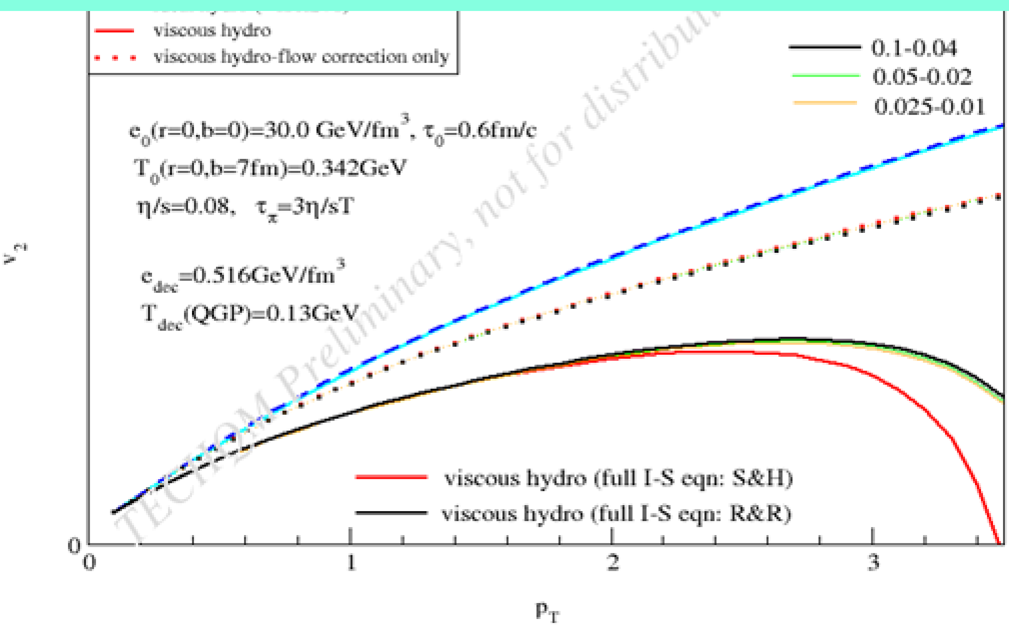
**Code Checking between Song & Heinz VISH2+1 and Romatschke Code (Nov.2007)**

ynamics

S L*




**Code Checking within TECHQM Collaboration (start from May, 2008)**




-Huovinen & Molnar: full I-S

QM08 talk: comparing the resul

-Chaudhuri: simplified I-S eq

**https://wiki.bnl.gov/TECHQM/index.php/Main_Page**

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**Issues:**

**- verification of the codes individually developed by different groups**

**-VISH2+1 (Song & Heinz) vs. Romatschke code: (Nov. 2007)**

**-VISH2+1 (Song & Heinz) vs. Dusling & Teaney code: (May, 2008- )**

# (2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

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**Issues:**

- verification of the codes individually developed by different groups
- **effects from different 2nd order formalisms**
  **simplified I-S eqn. vs. full I-S eqn., I-S eqn. vs. O-G eqn.**
- **effects from different EoS, systems sizes and freeze-out procedures**

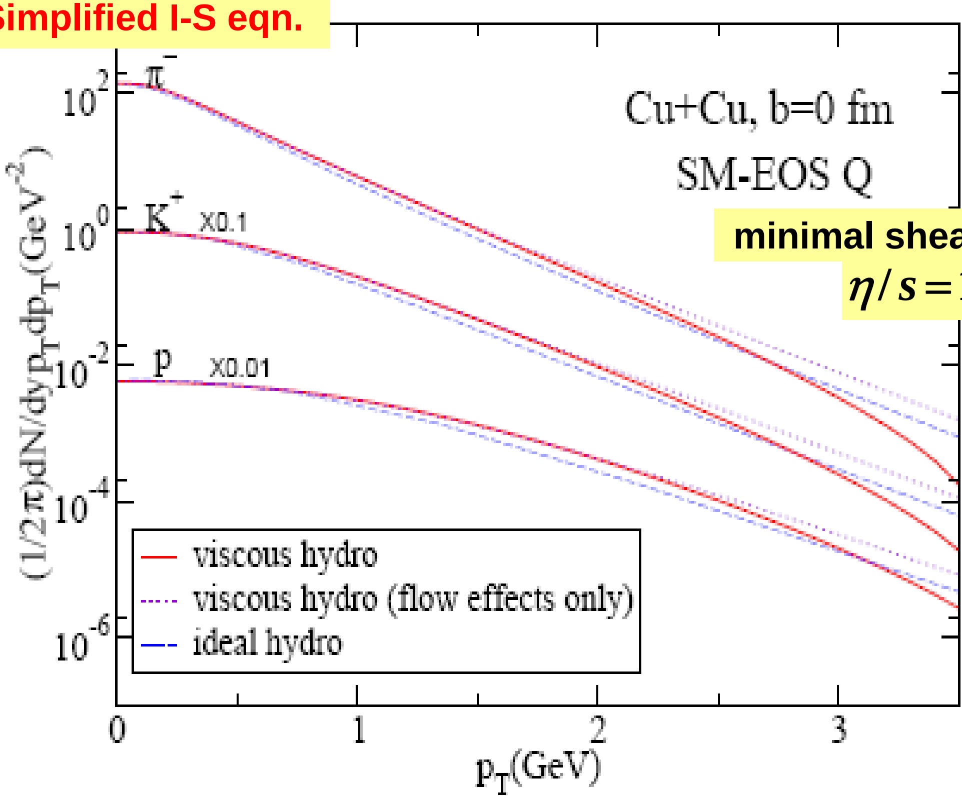
# Shear viscosity effects:

# Ideal hydro vs. viscous hydro

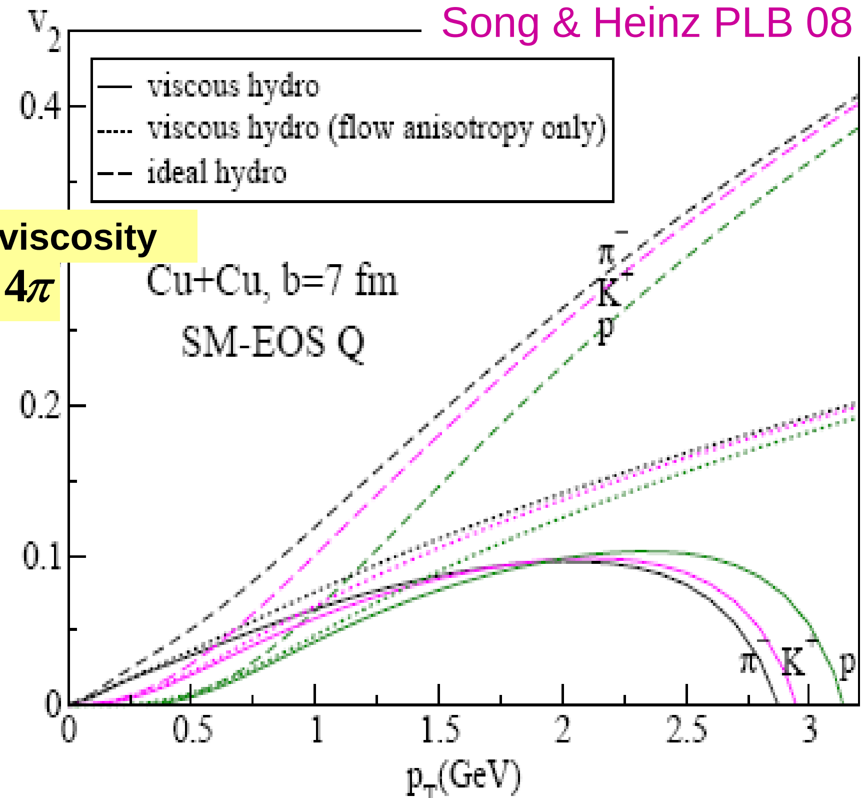
# Viscous vs. ideal hydro – spectra & elliptic flow

**Simplified I-S eqn.**

Song & Heinz PLB 08




**minimal shear viscosity**
$\eta/s = 1/4\pi$

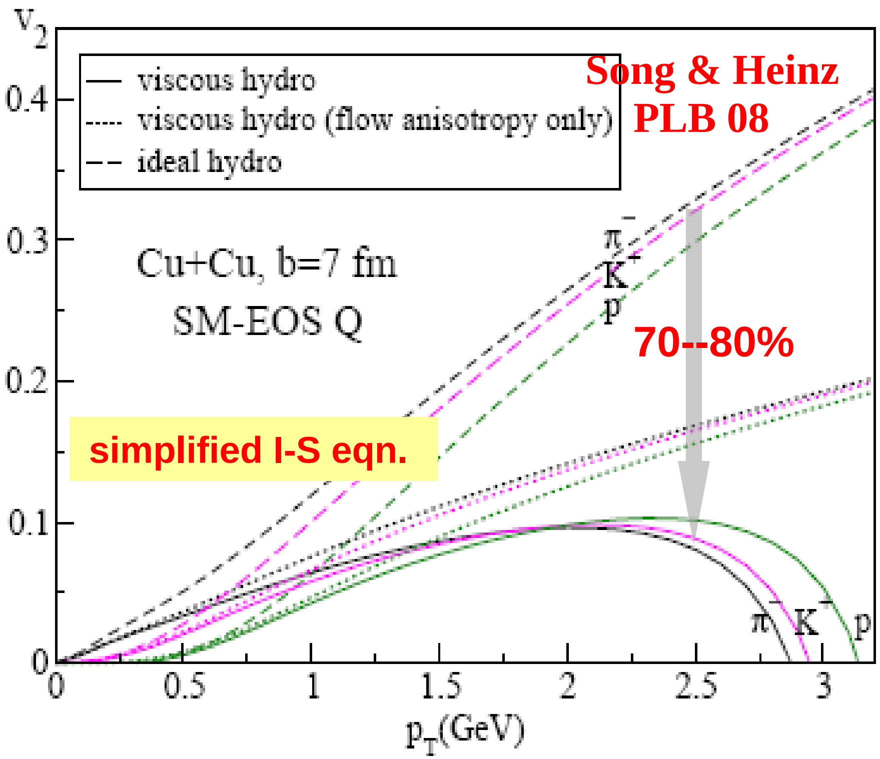


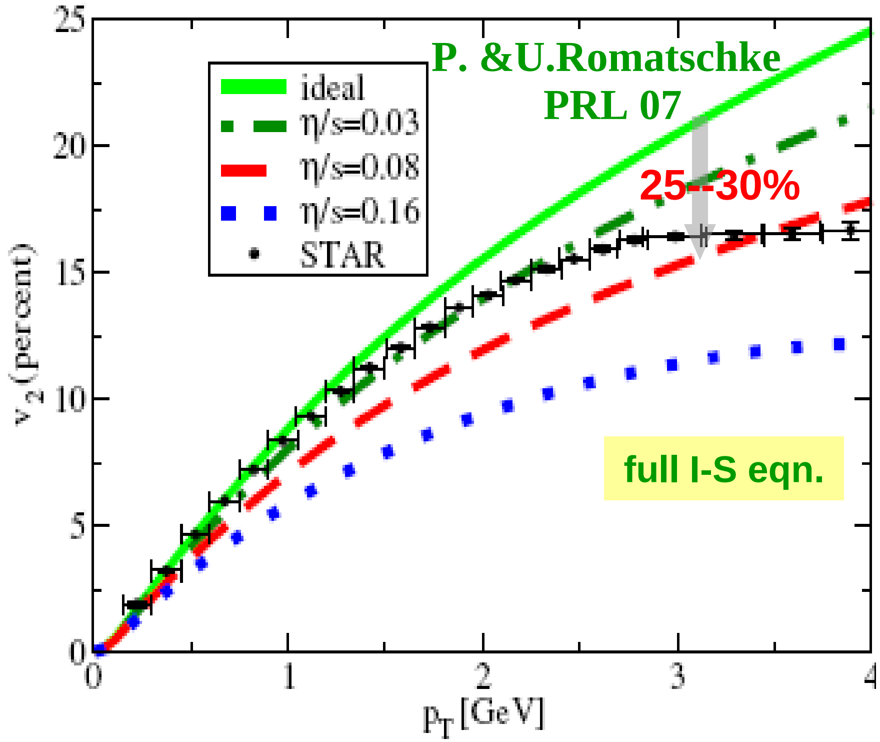

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{2\pi^3}\left[f_{eq}(x,p)+\delta f(x,p)\right] = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3} f_{eq}(x,p)\left(1+\frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e+p)(x)}\right)$$

-More radial flow, flatter spectra; elliptic flow is very sensitive to shear viscosity

-Both the evolution correction (viscous correction to flow in $f_0$) and spectra correction (viscous correction to $\delta f$ through $\pi^{\mu\nu}$) have significant effects on $v_2$.
For low $p_T$, the viscous correction to the flow is dominant.

# Comparison with Romatschke 07 results




- different systems & EOS: Cu+Cu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.

# Effect of using different I-S eqns.?

# Simplified I-S eqn. vs. full I-S eqn.:

simplified I-S eqn.: $$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right]$$

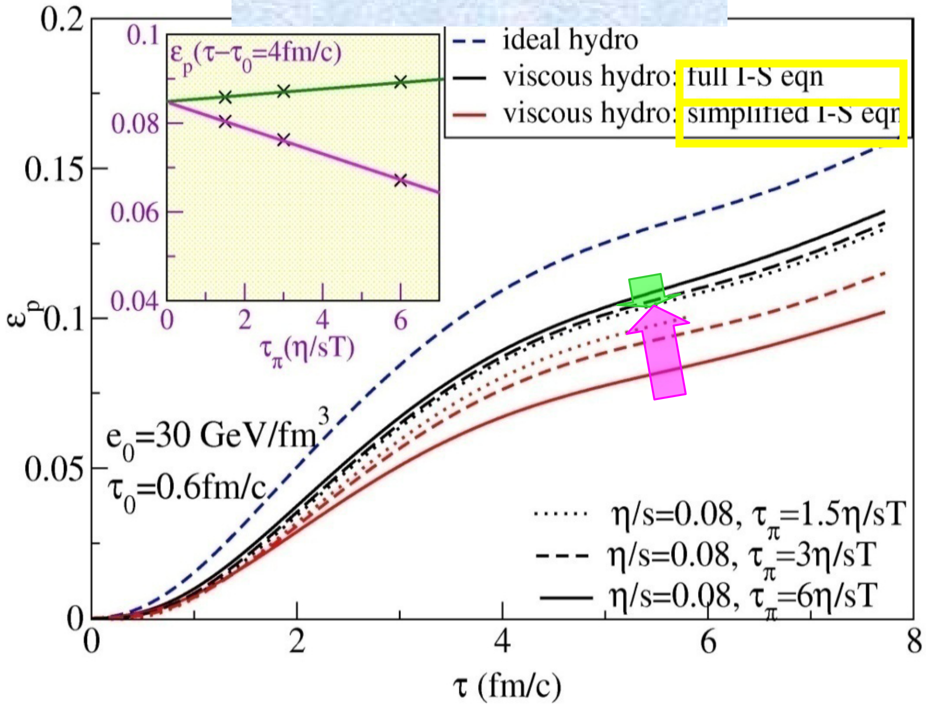
full I-S eqn.: $$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

important for preserving the conformal symmetry

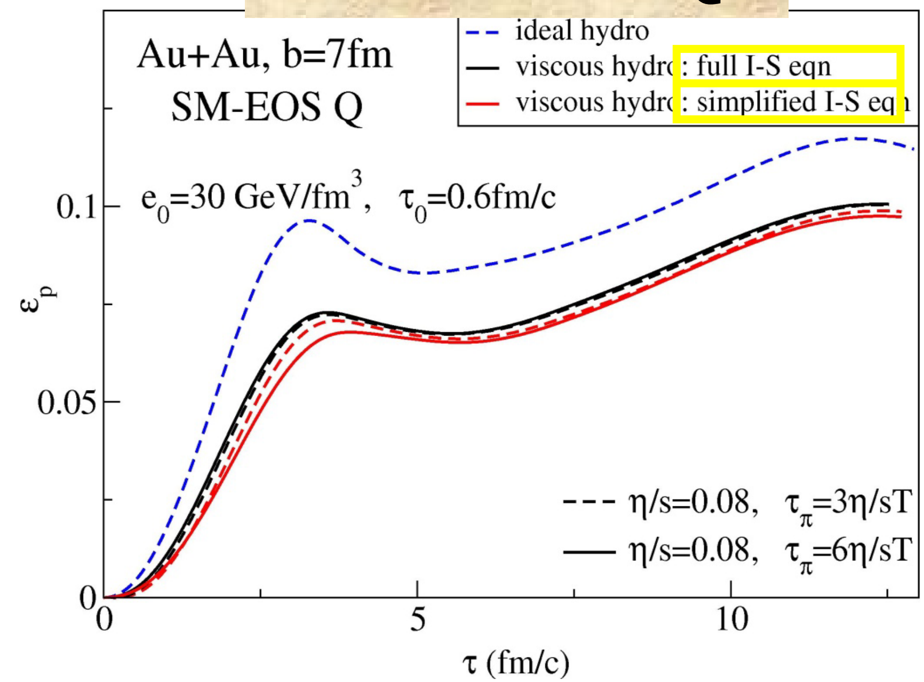
(Baier et al. '07)

# simplified I-S eqn. vs. full I-S eqn.

## EOS I




## SM-EOS Q



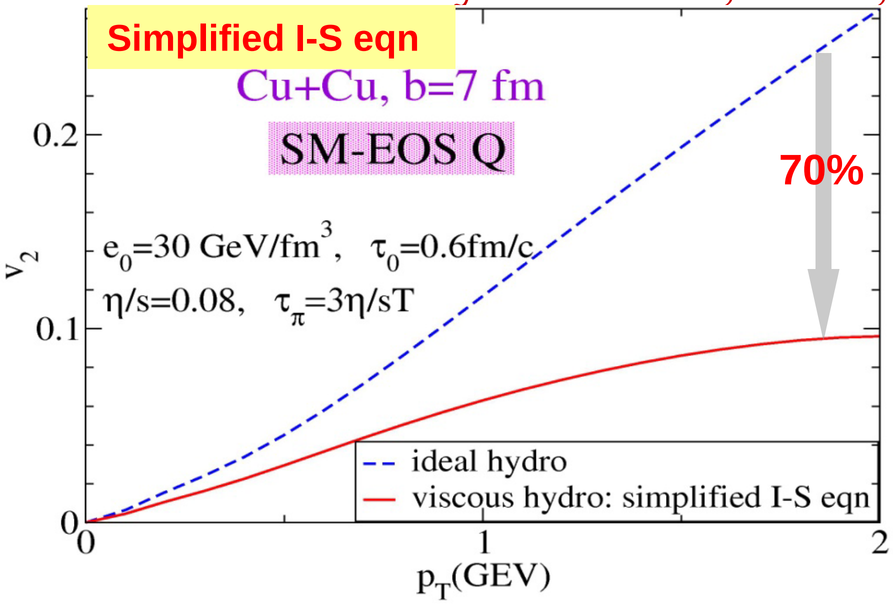

- for EOS I, the additional terms in full I-S eqn. bring 30-50% difference in the late-time momentum anisotropy and final $v_2$ suppression

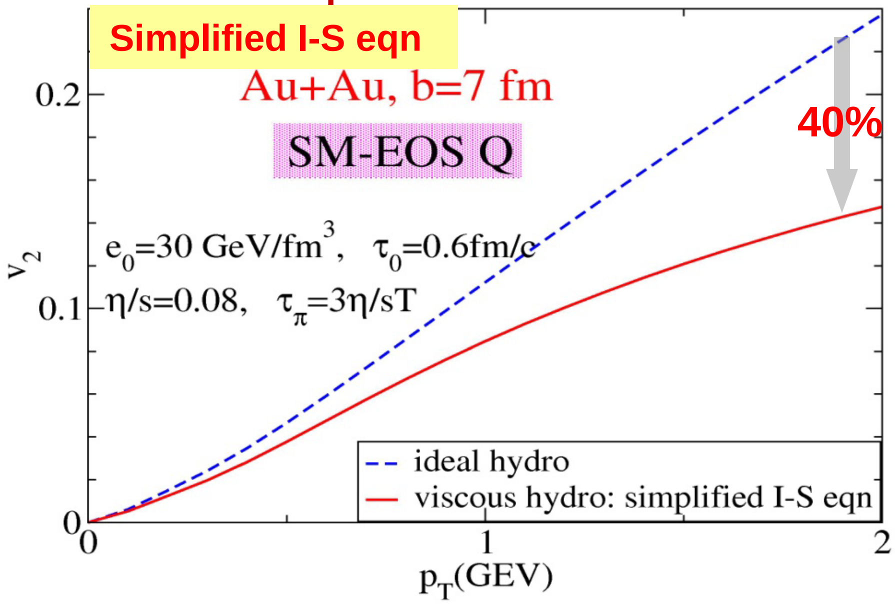
- for realistic EOS with a phase transition, the difference between simplified and full I-S for viscous v2 suppression are small

- numerical simulations also show that simplified I-S eqn. and full I-S eqn. **approach** the same Navier-Stokes limit as $\tau_\pi \to 0$, but the full I-S eqn. shows much weaker sensitivity to $\tau_\pi$

# System size effects to viscous $v_2$ suppression

system size, EOS, different I-S equations:

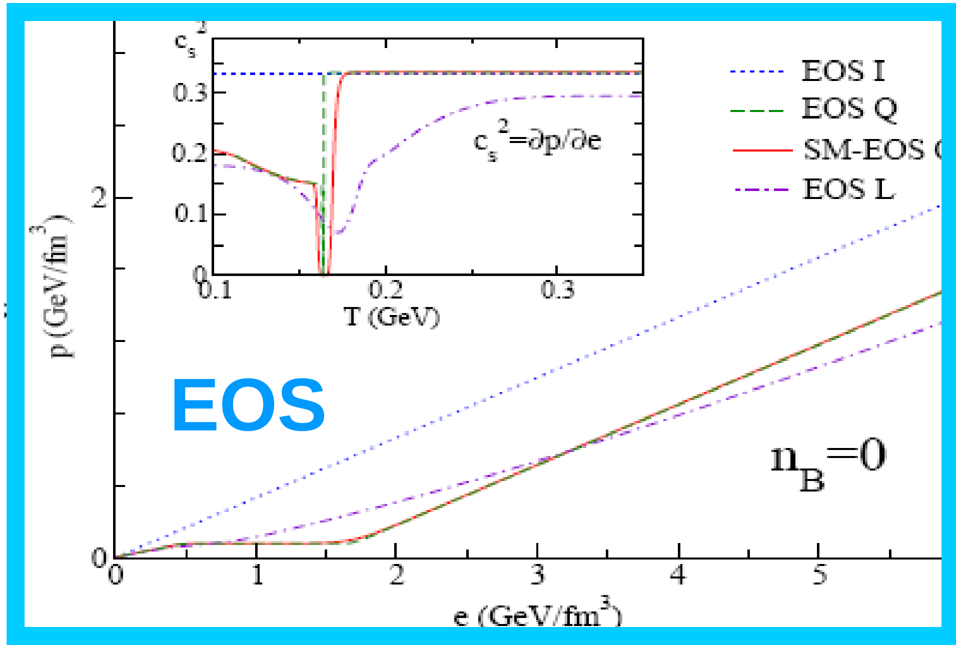


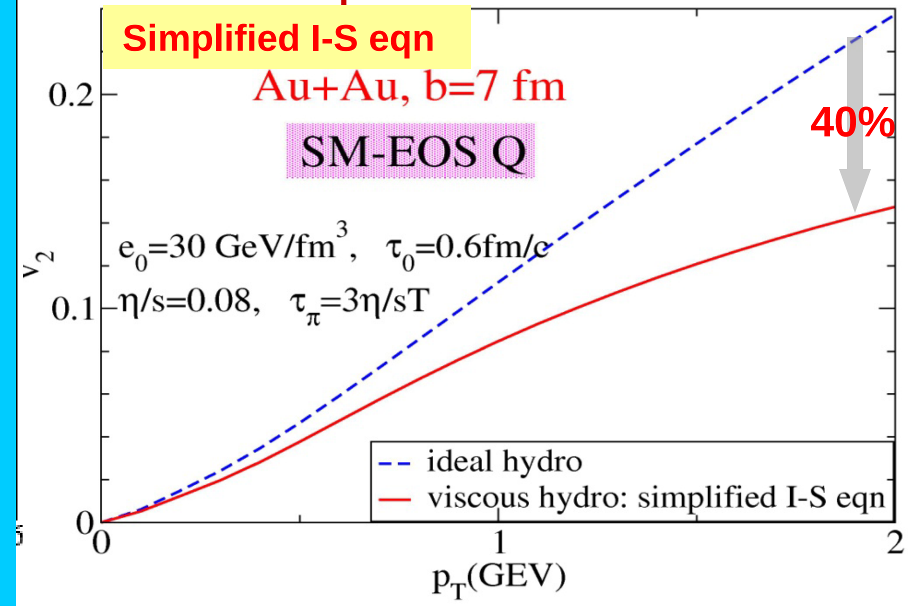



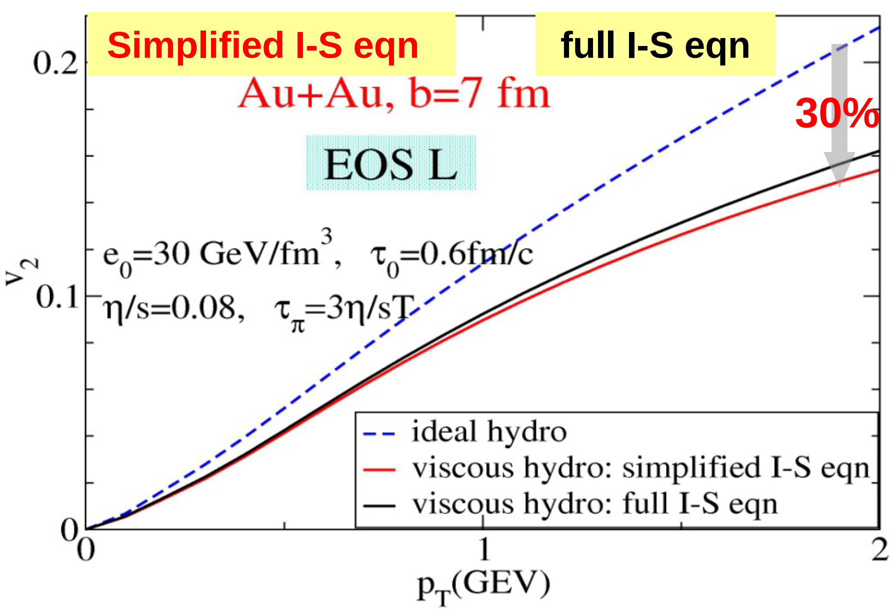

-system size: CuCu b=7fm vs. AuAu b=7fm:
~50-100% effect

# EOS effects to viscous $v_2$ suppression

…fferent I-S equations:



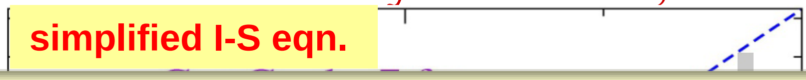

-system size: CuCu b=7fm vs. AuAu b=7fm: ~50-100% effect

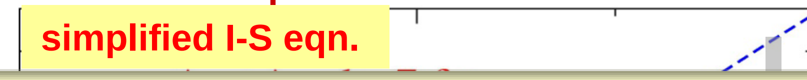
-EOS: SM-EOS Q vs. EOS L: ~25% effect

-different I-S eqns: simplified I-S eqn. vs. full I-S eqn.: ~5-10% effects (EOS Q and EOS L only )

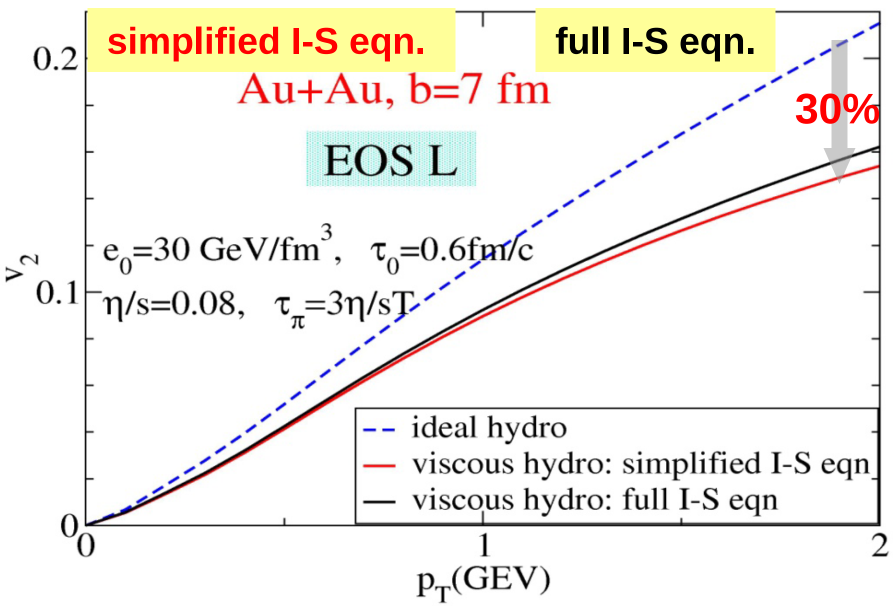
# Different contributions to the suppression of $v_2$

System size, EOS, different I-S equations:




Considering all of these effects, the final suppression of $v_2$ for Au+Au with EOS L and the full I-S eqn., for minimal shear viscosity $\eta / s = 0.08$, is ~25%, approaching the results of P. & U. Romatschke (PRL 99, 172301 (2007)).



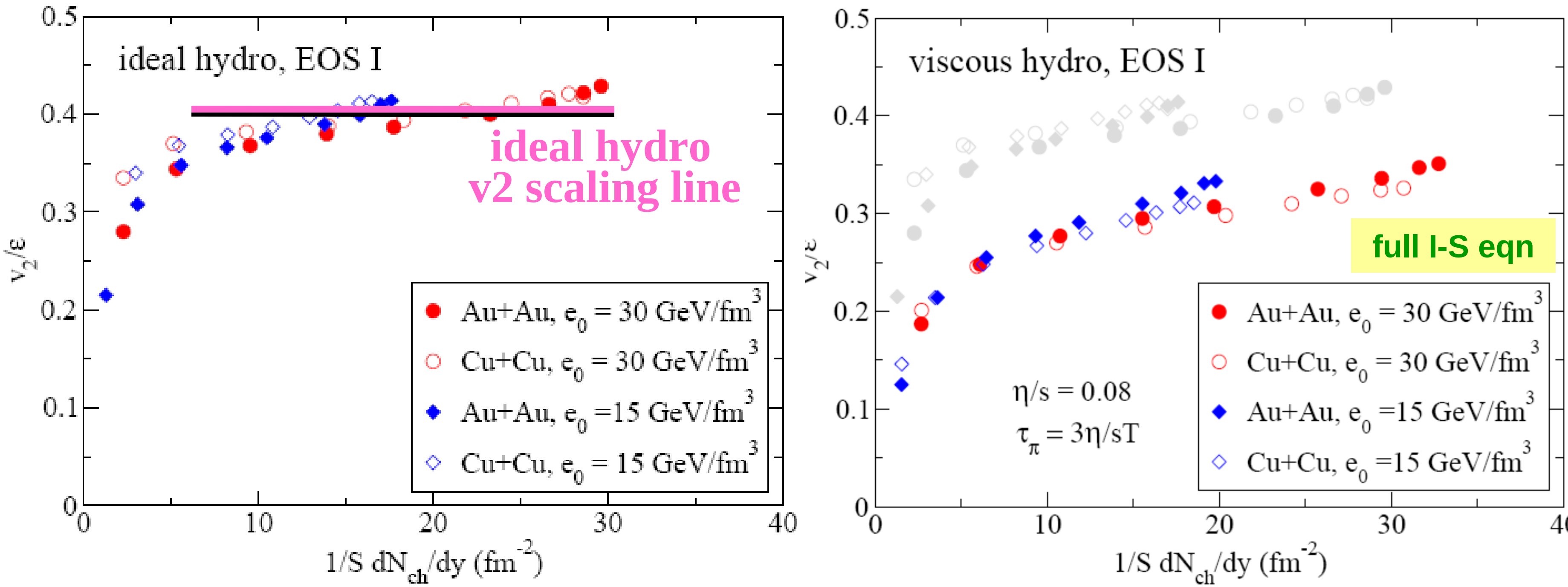

- system size: CuCu b=7fm vs. AuAu b=7fm: ~50-100% effect
- EoS: SM-EOS Q vs. EOS L: ~25% effect
- different I-S eqn.: simplified vs. full I-S eqn.: ~5-10% effect (EOS Q and EOS L only )

Comment: To extract QGP viscosity from exp. data by using viscous hydro, one needs a better description of EoS (Lattice EoS + chemical non-equil. HRG EoS)

# System size effects

# Multiplicity scaling of $v_2/\varepsilon$ EOS I

Preliminary




Ideal hydrodynamics: multiplicity scaling of $v_2/\varepsilon$ is weakly broken:

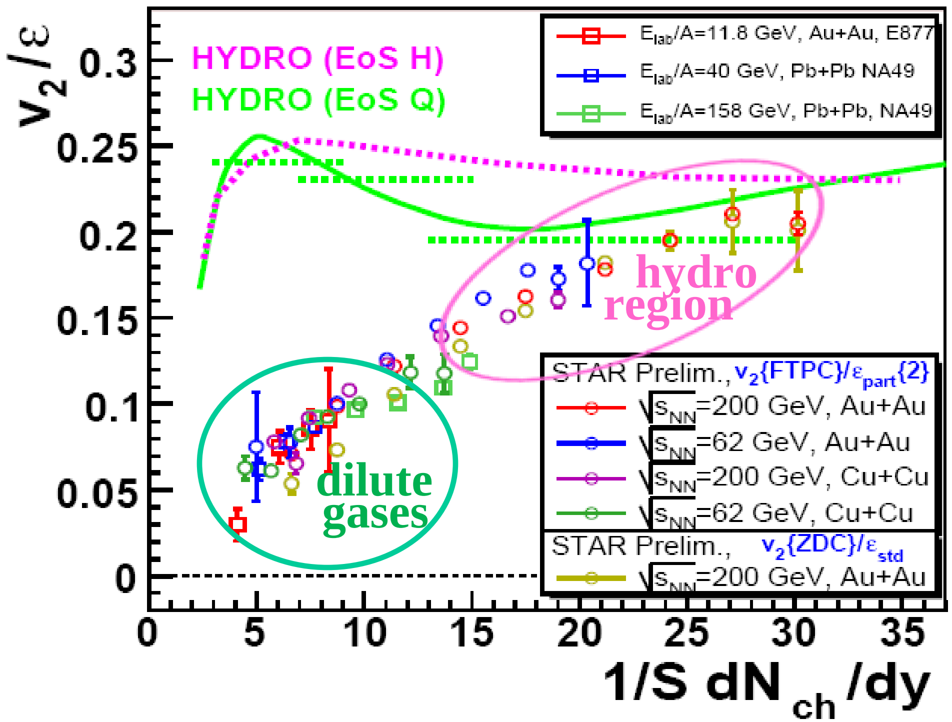
- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Cu+Cu and Au+Au systems are not identical after a rescaling

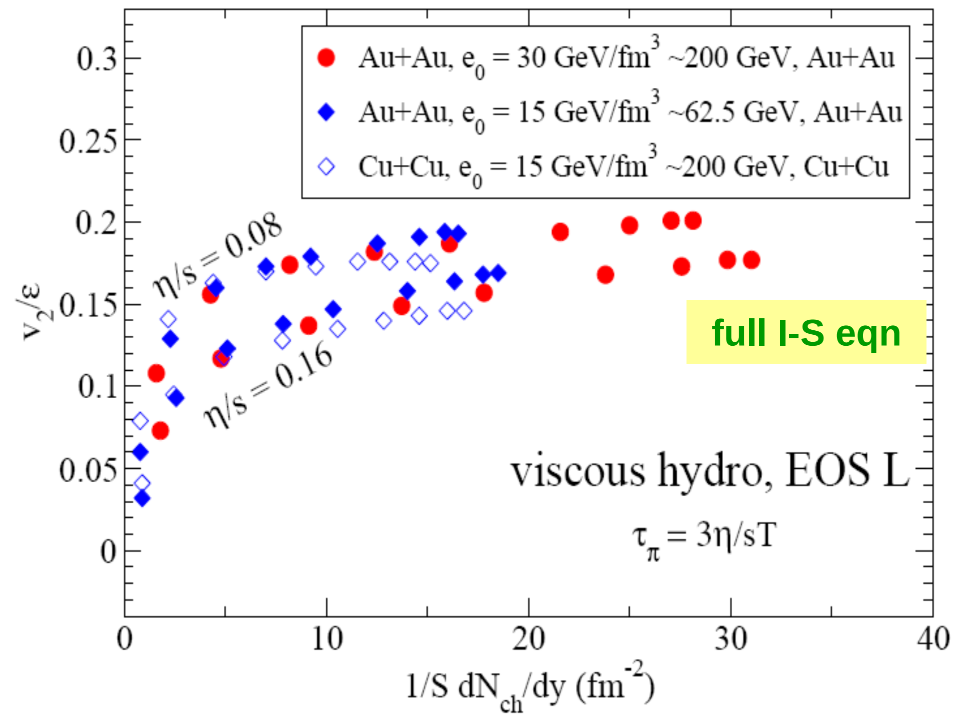
Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of $v_2/\varepsilon$ :

- for similar initial energy density, Cu+Cu curves are slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$ , the $e_0 = 15\,\mathrm{GeV/fm}^3$ curves are slightly above the $e_0 = 30\,\mathrm{GeV/fm}^3$ ones

Viscous effects are larger for smaller systems and lower collision energies

# Multiplicity scaling of $v_2/\varepsilon$ EOS L *Preliminary*




- experimental data show qualitatively similar fine ordering as viscous hydro prediction
- to reproduce slope of $v_2/\varepsilon$ vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: viscous hydro + hadron cascade
- the experimental $v_2/\varepsilon$ vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain $\eta/s$ (insensitive to Glauber-type vs. CGC initialization)
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for *T*-dependence of $\eta/s$, $\zeta/s$ near $T_c$, modeling hadronic phase with realistic cascade

# Summary and discussion

- $v_2$ is sensitive to $\eta/s$
- multiplicity scaling of $v_2/\varepsilon$ is a good candidate to extract the QGP viscosity:
  - larger viscous effects in smaller systems and at lower collision energies
  - multiplicity scaling of $v_2/\varepsilon$ is insensitive to Glauber model vs. CGC initialization.

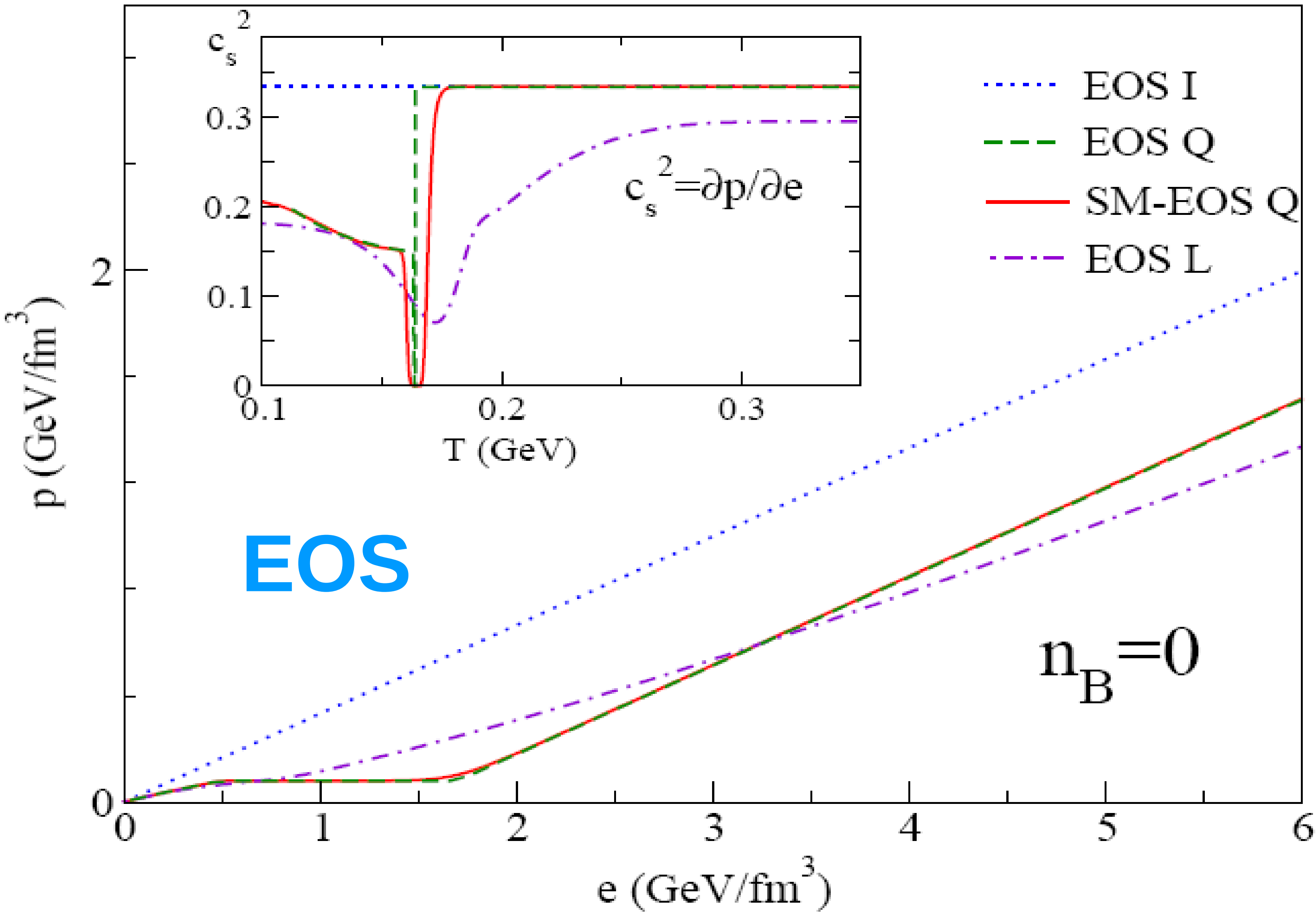
To extract QGP viscosity, one needs to consider at least the following aspects:

- a realistic EOS: EOS L vs. SM-EOS Q ~25% (for $v_2$ and $v_2/\varepsilon$ )
- initial conditions: CGC initialization vs. Glauber initialization ~15-30% (for $v_2$ )
- bulk viscosity: with vs. without bulk viscosity ~?%
- hadronic stage : viscous hydro+ hadron cascade in the furthure ?
- resolve the ambiguities between different 2nd order formalisms used by different groups to simulate causal viscous hydrodynamics
  a) simplified I-S eqn. (Song & Heinz 07-08) vs. full I-S eqn. (P.&U.Romatschke)
  b) I-S formalism vs. Öttinger-Grmela (O-G) formalism (Dusling & Teaney ) ?

…… ……

# Thank You

# EOS



p (GeV/fm^3)
e (GeV/fm^3)
EOS I
EOS Q
SM-EOS Q
EOS L
c_s^2
c_s^2=∂p/∂e
T (GeV)
EOS
n_B=0